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Non-Standard Analysis in Electrical Engineering. Complex Circuits with Ideal Reactive Elements

The article presents the application of concepts and methods of non-standard analysis to problems of theoretical electrical engineering. It is substantiated that standard methods of electrical engineering are not effective enough, because they are excessively complex or even unsuitable for calculating DC electrical circuits containing ideal inductances and capacitances. This problem arises because for direct current (with zero frequency) the inductive resistance is zero, and the capacitive resistance goes to infinity. As a result, when using standard methods to calculate such electrical circuits, it is necessary to solve uncertainty expressions of the type $\frac{0}{0}$ or $\frac{\infty}{\infty}$, which creates difficulties within the framework of traditional mathematical analysis. Given the above difficulties, it is proposed to replace the classical mathematical analysis with a non-standard one. In this approach, the frequency of the direct current is considered not as zero, but as an infinitely small value α. This approach makes it possible to determine the reactance of inductive elements as αL and the reactance of capacitive elements as- $\frac{1}{\alpha C}$. This allows to successfully apply all standard methods of theoretical electrical engineering and avoid the need to work with indefinite expressions. The article provides specific examples of the analysis of complex direct current circuits with ideal inductances and capacitances.

Keywords: complex direct current circuits, non-standard analysis, electrical engineering, ideal reactive element

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Introduction

Non-standard analysis became widely used in the middle of the last century, which was made possible by the new axiomatics of mathematical analysis proposed at that time. Nowadays, non-standard analysis methods are quite often used in various fields of science and technology. When solving various technical and scientific problems, in some cases it is necessary to reveal uncertainties of the type $\frac{0}{0}$ or $\frac{\infty}{\infty}$, which is extremely difficult when using classical methods of analysis [1]. In such cases, it is advisable to use ideas and methods of non-standard analysis, which consist in the direct use of infinitely small and large numbers [2, 3]. In [4], the axiomatics of mathematical analysis are considered, which is based on the set of hyperreal numbers, which contains, in addition to standard numbers, also non-standard (infinitely small, infinitely large, their combinations with standard numbers) numbers. The use of non-standard analysis methods in identifying the internal parameters of electric motors, which in many cases cannot be implemented using traditional methods, is promising [5, 6]. Also, non-standard analysis methods can be used for mathematical modeling of electromechanical objects [7–9], mathematical modeling and calculation of measuring channels of electrical and non-electrical physical quantities [10–12], calculation of building structures [13, 14], analysis of biological [15], chemical [16] processes, as well as in medicine [17, 18]. The use of non-standard analysis in theoretical electrical engineering is especially important. Traditionally, calculation methods based on Ohm's and Kirchhoff's laws are used to analyze DC electrical circuits, but there are a number of tasks for which the use of these methods is practically impossible. Therefore, it is relevant to use the mathematical apparatus of non-standard analysis, which will allow using known unified methods for calculating such circuits. For example, this applies to the calculation of DC circuits with ideal reactive elements [19], as well as the analysis of transient processes in second-order electrical circuits with violation of switching laws [20]. This article discusses the use of non-standard analysis methods for complex circuits with ideal reactive elements.

Formulation of the problem

Calculation of DC circuits is usually carried out using traditional methods based on Ohm's and Kirchhoff's laws. However, such standard approaches are unsuitable for certain specific cases, in particular for complex electrical circuits containing ideal reactive elements. The complexity arises from the fact that at DC the resistance of an ideal inductance is zero, and the resistance of an ideal capacitance tends to infinity, which in this case makes it impossible to calculate complex circuits. In some cases, such problems can be solved by calculating using the energy characteristics of reactive elements, which significantly complicates the calculations, especially for complex circuits. In view of this, the application of the mathematical apparatus of non-standard analysis, which allows the use of unified methods for calculating such circuits, is relevant and promising. The paper considers non-standard analysis of complex DC circuits with ideal reactive elements. The aim of the article is to define a class of non-standard electrical engineering problems for the analysis of complex DC circuits with ideal inductances and capacitances, as well as to extend non-standard analysis methods to a wider class of problems involving the calculation of complex circuits with ideal inductances and capacitances.

Research results

Let R denote the ordered set of real numbers, and we will call the number α an infinitely small number if and only if

$$\forall r \in R(\alpha < r). \tag{1}$$

An infinitely large number will be called the number $\beta = \frac{1}{\alpha}$, for this case we can write

$$\forall r \in R(\beta > r). \tag{2}$$

Infinitesimal numbers of the first, second, third, and k-th order are defined, respectively, as $\alpha > \alpha^2 > \alpha^3 > \alpha^k$, and infinitely large numbers of the first, second, third, and k-th order are defined, respectively, as $\beta < \beta^2 < \beta^3 < \beta^k$.

All the rules of algebra apply to infinitely small and infinitely large numbers, namely addition, subtraction, multiplication, division, exponentiation, and theorems (commutativity, associativity, etc.). Infinitely small and infinitely large numbers, together with the real numbers $r \in R$, form an ordered set of hyperreal numbers $*R$, the real numbers $r \in R$ are called standard or Archimedean in contrast to the non-standard numbers $*r \in *R$. In the following, the notation $\approx$. ll mean the equivalence of two non-standard numbers.

For standard numbers m and n, the following relations are valid:

$$\frac{m\alpha}{n\alpha} = \frac{m}{n},\ \frac{m\alpha}{n} = \frac{m}{n}\alpha,\ m\alpha + n \approx n,\ m\alpha^k + n \approx n,\ \sin\alpha \approx \alpha,\ \cos\alpha \approx 1. \tag{3}$$

Not only the set of real numbers, but also the set of complex numbers has the same structure, based on this we can write:

$$m\alpha + jn \approx jn,\ m\beta + jn \approx m\beta,\ m + jn\alpha \approx m,\ m + jn\beta \approx jn\beta. \tag{4}$$

Since, in this case, the DC circuit is considered as a sinusoidal current circuit, the cyclic frequency of which is zero, the symbolic method can be used to solve such a problem, provided that $\omega = \alpha$. Assuming $\omega = \alpha$, for the complex resistance of the inductance we can write

$$\underline{Z}_L \approx j\alpha L, \tag{5}$$

and for the complex capacitance resistance

$$\underline{Z}_C \approx \frac{1}{j\alpha C}, \tag{6}$$

As a first example, consider the complex electrical circuit with three ideal inductances, two of which are magnetically coupled. First, consider the matched connection of ideal inductances, as shown in Fig. 1.

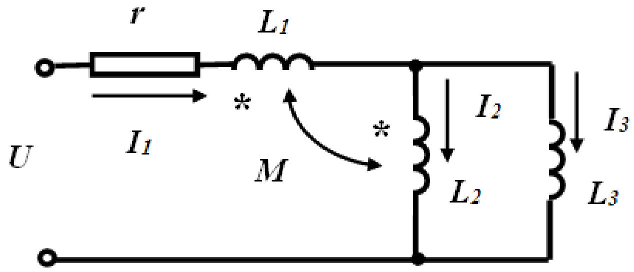

Figure 1. Electrical circuit with three ideal inductances and magnetic coupling between two of them (matched switching)

For the given circuit, the equations according to Kirchhoff's laws have the form:

$$\underline{I}_1 - \underline{I}_2 - \underline{I}_3 = 0, \tag{7}$$

$$\underline{I}_1 \left(r + j\alpha L_1 + j\alpha M \right) + \underline{I}_2 \left(j\alpha L_2 + j\alpha M \right) = U, \tag{8}$$

$$\underline{I}_3 j\alpha L_3 = \underline{I}_2 j\alpha L_2 + \underline{I}_1 j\alpha M. \tag{9}$$

Let perform equivalent transformations for equation (8).

$$\underline{I}_1 \left(r + j\alpha L_1 + j\alpha M \right) + \underline{I}_2 \left(j\alpha L_2 + j\alpha M \right) \approx I_1 r = U. \tag{10}$$

From expression (10) it follows:

$$\underline{I}_1 = \frac{U}{r}. \tag{11}$$

Thus, new equations were obtained in accordance with Kirchhoff's laws:

$$\frac{U}{r} - \underline{I}_2 - \underline{I}_3 = 0, \tag{12}$$

$$\underline{I}_3 j\alpha L_3 = \underline{I}_2 j\alpha L_2 + \frac{U}{r} j\alpha M. \tag{13}$$

Let’s determine the current $\underline{I}_3$ from the equation (12) and substitute it into the second equation.

$$\underline{I}_3 = \frac{U}{r} - \underline{I}_2, \tag{14}$$

$$\left(\frac{U}{r} - \underline{I}_2 \right) j\alpha L_3 = \underline{I}_2 j\alpha L_2 + \frac{U}{r} j\alpha M. \tag{15}$$

From these equations it follows

$$\frac{U}{r} j\alpha L_3 - \underline{I}_2 j\alpha L_3 = \underline{I}_2 j\alpha L_2 + \frac{U}{r} j\alpha M, \tag{16}$$

$$\frac{U}{r} \left(j\alpha L_3 - j\alpha M \right) = \underline{I}_2 \left(j\alpha L_3 + j\alpha L_2 \right), \tag{17}$$

$$\underline{I}_2 = \frac{\frac{U}{r} \left(j\alpha L_3 - j\alpha M \right)}{\left(j\alpha L_3 + j\alpha L_2 \right)} = \frac{U \left(L_3 - M \right)}{r \left(L_3 + L_2 \right)}, \tag{18}$$

$$\underline{I}_3 = \frac{U}{r} - \underline{I}_2 = \frac{U}{r} - \frac{U \left(L_3 - M \right)}{r \left(L_3 + L_2 \right)} = \frac{U \left(L_2 + M \right)}{r \left(L_3 + L_2 \right)}. \tag{19}$$

As an example, let perform numerical calculations of the complex circuit shown in Figure 1 for three characteristic cases of the ratio between the inductances L_1, L_2, L_3 and the mutual inductance M, setting the following parameters: $U = 30$ V, $r = 10$ Ohm, $L_1 = 0.2$ H, $L_2 = 0.1$ H, $L_3 = 0.12$ H.

For the first characteristic case $M = 0.05$ H, i.e. $M < L_2$. As a result of the calculation, we obtain $\underline{I}_2 = 0.955$ A, and $\underline{I}_3 = 2.045$ A.

For the second characteristic case $M = 0.12$ H, i.e. $M = L_3$. In this case, all the current flows in the third coil ($\underline{I}_3 = 3$ A), and in the second coil it disappears ($\underline{I}_2 = 0$ A).

The most interesting is the third characteristic case, $M = 0.14$ H, $L_3 < M < \sqrt{L_1 L_2}$. Here, the so-called "false capacitance effect" is observed, when the current in the third coil ($\underline{I}_3 = 3.273$ A) exceeds the input current, and in the second coil $\underline{I}_2 = -0.273$ A the current changes its direction.

Let consider the case of counter-switching of inductors (Fig. 2).

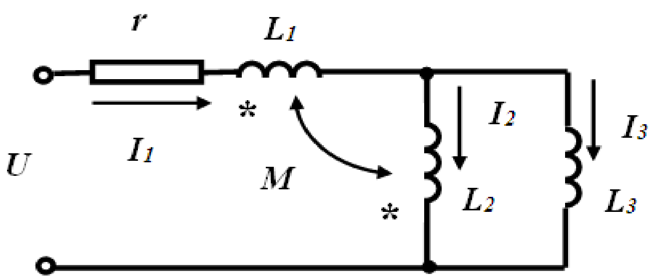

Figure 2. A circuit with three ideal inductances and magnetic coupling between two of them (counter-switching)

For the given circuit, the equations according to Kirchhoff's laws have the form:

$$\underline{I}_1 - \underline{I}_2 - \underline{I}_3 = 0, \tag{20}$$

$$\underline{I}_1 \left(r + j\alpha L_1 - j\alpha M \right) + \underline{I}_2 \left(j\alpha L_2 - j\alpha M \right) = U, \tag{21}$$

$$\underline{I}_3 j\alpha L_3 = \underline{I}_2 j\alpha L_2 - \underline{I}_1 j\alpha M. \tag{22}$$

Let perform equivalent transformations for equation (21).

$$\underline{I}_1 \left(r + j\alpha L_1 - j\alpha M \right) + \underline{I}_2 \left(j\alpha L_2 - j\alpha M \right) \approx \underline{I}_1 r = U. \tag{23}$$

From expression (23) it follows:

$$\underline{I}_1 = \frac{U}{r}. \tag{24}$$

Thus, new equations were obtained in accordance with Kirchhoff's laws:

$$\frac{U}{r} - \underline{I}_2 - \underline{I}_3 = 0, \tag{25}$$

$$\underline{I}_3 j\alpha L_3 = \underline{I}_2 j\alpha L_2 - \frac{U}{r} j\alpha M. \tag{26}$$

Let's determine the current $\underline{I}_3$ from the equation (25) and substitute it into the second equation.

$$\underline{I}_3 = \frac{U}{r} - \underline{I}_2, \tag{27}$$

$$\left(\frac{U}{r} - \underline{I}_2 \right) j\alpha L_3 = \underline{I}_2 j\alpha L_2 - \frac{U}{r} j\alpha M. \tag{28}$$

From these equations it follows

$$\frac{U}{r} j\alpha L_3 - \underline{I}_2 j\alpha L_3 = \underline{I}_2 j\alpha L_2 - \frac{U}{r} j\alpha M \,, \tag{29}$$

$$\frac{U}{r}\left(j\alpha L_3 + j\alpha M \right) = \underline{I}_2 \left(j\alpha L_3 + j\alpha L_2 \right), \tag{30}$$

$$\underline{I}_2 = \frac{\frac{U}{r}\left(j\alpha L_3 + j\alpha M \right)}{\left(j\alpha L_3 + j\alpha L_2 \right)} = \frac{U\left(L_3 + M\right)}{r\left(L_3 + L_2\right)}, \tag{31}$$

$$\underline{I}_3 = \frac{U}{r} - \underline{I}_2 = \frac{U}{r} - \frac{U\left(L_3 + M\right)}{r\left(L_3 + L_2\right)} = \frac{U\left(L_2 - M\right)}{r\left(L_3 + L_2\right)} \,. \tag{32}$$

As an example, let perform numerical calculations of the complex circuit shown in Figure 2 for three characteristic cases of the ratio between the inductances L_1, L_2, L_3 and the mutual inductance M, setting the following parameters: $U = 30$ V, $r = 10$ Ohm, $L_1 = 0.2$ H, $L_2 = 0.1$ H, $L_3 = 0.12$ H.

For the first characteristic case $M = 0.05$ H, i.e. $M < L_2$. As a result of the calculation, we obtain $\underline{I}_2 = 2.318$ A, and $\underline{I}_3 = 0.682$ A.

For the second characteristic case $M = 0.1$ H, i.e. $M = L_2$. In this case, all the current flows in the second coil ($\underline{I}_2 = 3$ A), and in the third coil it disappears ($\underline{I}_3 = 0$ A).

The most interesting is the third characteristic case, $M = 0.14$ H, $L_2 < M < \sqrt{L_1 L_2}$. Here, the so-called "false capacitance effect" is observed, when the current in the second coil ($\underline{I}_2 = 3.545$ A) exceeds the input current, and in the third coil $\underline{I}_3 = -0.545$ A the current changes its direction.

It should be noted that this task is very difficult to solve without the help of non-standard analysis methods, and for the next task it is practically impossible.

Let consider the second example, in which we determine the currents in all branches of a complex electrical circuit with ideal inductances and resistors, which is shown in Figure 3.

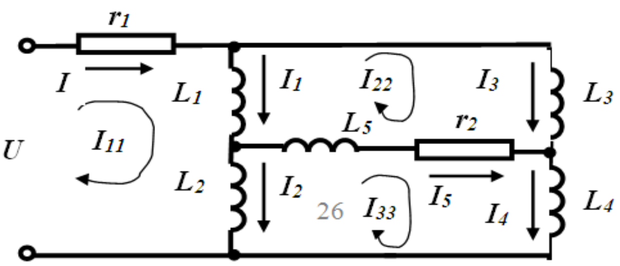

Figure 3. Complex electrical circuit with ideal inductances and resistors

Let the circuit shown in Figure 3 have the following parameters: $U = 100$ V, $r_1 = 10$ Ohm, $r_2 = 20$ Ohm, $L_1 = 0.2$ H, $L_2 = 0.15$ H, $L_3 = 0.1$ H, $L_4 = 0.05$ H, $L_5 = 0.025$ H. Let calculate this circuit using the loop current method. The equations for the loop current method have the form:

$$\underline{I}_{11}\underline{Z}_{11} + \underline{I}_{22}\underline{Z}_{12} + \underline{I}_{33}\underline{Z}_{13} = U \,, \tag{33}$$

$$\underline{I}_{11}\underline{Z}_{21} + \underline{I}_{22}\underline{Z}_{22} + \underline{I}_{33}\underline{Z}_{23} = 0 \,, \tag{34}$$

$$\underline{I}_{11}\underline{Z}_{31} + \underline{I}_{22}\underline{Z}_{32} + \underline{I}_{33}\underline{Z}_{33} = 0 \,. \tag{35}$$

Let find the self and common resistances of the contours:

$$\underline{Z}_{21} = -j\alpha L_1 ,\ \underline{Z}_{23} = -r_2 - j\alpha L_5 ,\ \underline{Z}_{31} = -j\alpha L_2 , \tag{36}$$

$$\underline{Z}_{22} = j\alpha L_1 + j\alpha L_3 + j\alpha L_5 + r_2 \,, \tag{37}$$

$$\underline{I}_{33} = j\alpha L_2 + j\alpha L_4 + j\alpha L_5 + r_2 \,. \tag{38}$$

Let first consider equation (33), substituting into it expressions for the self and common resistances.

$$\underline{I}_{11}\left(r_1 + j\alpha L_1 + j\alpha L_2\right) + \underline{I}_{22}\left(-j\alpha L_1\right) + \underline{I}_{33}\left(-j\alpha L_2\right) = U \ . \quad (39)$$

Since only the coefficient at $\underline{I}_{11}$ is not infinitely small, we perform equivalent transformations:

$$\underline{I}_{11}\left(r_1 + j\alpha L_1 + j\alpha L_2\right) + \underline{I}_{22}\left(-j\alpha L_1\right) + \cdot_{33}\left(-j\alpha L_2\right) \approx \underline{I}_{11} r_1 = U \ . \quad (40)$$

From equation (40) it follows

$$\underline{I}_{11} = \underline{I} = \frac{U}{r_1} . \quad (41)$$

Similarly, we perform equivalent transformations for equation (34).

$$\frac{U}{r_1}\left(-j\alpha L_1\right) + \underline{I}_{22}\left(j\alpha L_1 + j\alpha L_3 + j\alpha L_5 + r_2\right) + \underline{I}_{33}\left(-r_2 - j\alpha L_5\right) \approx r_2 \underline{I}_{22} - r_2 \underline{I}_{33} = 0. \quad (42)$$

From equation (42) we obtain

$$\underline{I}_{22} = \underline{I}_{33} , \quad (43)$$

and

$$\underline{I}_5 = \underline{I}_{33} - \underline{I}_{22} = 0 . \quad (44)$$

Let perform equivalent transformations for equation (35).

$$\frac{U}{r_1}\left(-j\alpha L_2\right) + \underline{I}_{22}\left(-r_2 - j\alpha L_5\right) + \underline{I}_{33}\left(j\alpha L_2 + j\alpha L_4 + j\alpha L_5 + r_2\right) \approx -r_2 \frac{n!}{r!(n-r)!_{22}} + r_2 \underline{I}_{33} = 0. \quad (45)$$

It is obvious that

$$\underline{I}_{22} = \underline{I}_{33} . \quad (46)$$

Since there is no current in the inductance L_5, the task is significantly simplified, so we can write

$$\underline{I}_1 = \underline{I}_2 = \frac{U\left(L_3 + L_4\right)}{\left(L_1 + L_2 + L_3 + L_4\right) r_1} = 3\,\text{A}, \quad (47)$$

$$\underline{I}_3 = \underline{I}_4 = \frac{U\left(L_1 + L_2\right)}{\left(L_1 + L_2 + L_3 + L_4\right) r_1} = 7\ \text{A}. \quad (48)$$

Let's consider the third example, we will determine in a DC branched circuit with ideal capacitances (Fig. 4) the voltages on the capacitors C_1, C_2, C_3, the circuit parameters: $U = 100$ V, $C_1 = 200$ μF, $C_2 = 150$ μF, $C_3 = 100$ μF.

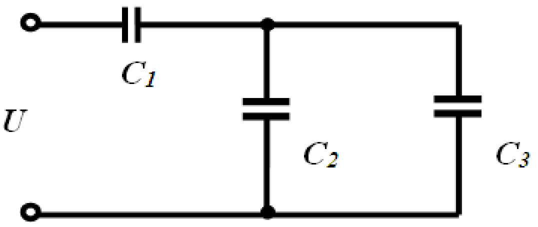

Figure 4. Branched circuit with ideal capacitances

Let find the total complex resistance of the circuit

$$\underline{Z}_{вх} \approx \frac{1}{j\alpha C_1} + \frac{\frac{1}{j\alpha C_2} \cdot \frac{1}{j\alpha C_3}}{\frac{1}{j\alpha C_2} + \frac{1}{j\alpha C_3}} = \frac{1}{j\alpha C_1} + \frac{\frac{1}{\left(j\alpha C_2\right)\left(j\alpha C_3\right)}}{\frac{j\alpha C_2 + j\alpha C_3}{\left(j\alpha C_2\right)\left(j\alpha C_3\right)}} =$$

$$= \frac{1}{j\alpha C_1} + \frac{1}{j\alpha C_2 + j\alpha C_3} = \frac{j\alpha C_1 + j\alpha C_2 + j\alpha C_3}{j\alpha C_1\left(j\alpha C_2 + j\alpha C_3\right)} = \frac{C_1 + C_2 + C_3}{j\alpha C_1\left(C_2 + C_3\right)}, \tag{49}$$

then the current flowing through the capacitance C_1

$$\underline{I}_{C_1} = \frac{U}{\underline{Z}_{\text{вх}}} = \frac{Uj\alpha C_1\left(C_2 + C_3\right)}{C_1 + C_2 + C_3}. \tag{50}$$

The currents flowing through the capacitors C_2, C_3 are found as

$$\underline{I}_{C_2} = \underline{I}_{C_1} \frac{\dfrac{1}{j\alpha C_3}}{\dfrac{1}{j\alpha C_2} + \dfrac{1}{j\alpha C_3}} = \frac{Uj\alpha C_1\left(C_2 + C_3\right)}{C_1 + C_2 + C_3} \cdot \frac{\left(j\alpha C_2\right)}{j\alpha C_2 + j\alpha C_3} = \frac{Uj\alpha C_1 C_2}{C_1 + C_2 + C_3}, \tag{51}$$

$$\underline{I}_{C_3} = \underline{I}_{C_1} \frac{\dfrac{1}{j\alpha C_2}}{\dfrac{1}{j\alpha C_2} + \dfrac{1}{j\alpha C_3}} = \frac{Uj\alpha C_1\left(C_2 + C_3\right)}{C_1 + C_2 + C_3} \cdot \frac{\left(j\alpha C_3\right)}{j\alpha C_2 + j\alpha C_3} = \frac{Uj\alpha C_1 C_3}{C_1 + C_2 + C_3}. \tag{52}$$

The voltages on the capacitors are determined by the following expressions:

$$U_{C_1} = \underline{I}_{C_1} \frac{1}{j\alpha C_1} = \frac{Uj\alpha C_1\left(C_2 + C_3\right)}{C_1 + C_2 + C_3} \cdot \frac{1}{j\alpha C_1} = \frac{U\left(C_2 + C_3\right)}{C_1 + C_2 + C_3} = 42.857 \text{ V}, \tag{53}$$

$$U_{C_2} = \underline{I}_{C_2} \frac{1}{j\alpha C_2} = \frac{Uj\alpha C_1 C_2}{C_1 + C_2 + C_3} \cdot \frac{1}{j\alpha C_2} = \frac{UC_1}{C_1 + C_2 + C_3} = 57.143 \text{ V}, \tag{54}$$

$$U_{C_3} = \underline{I}_{C_3} \frac{1}{j\alpha C_3} = \frac{Uj\alpha C_1 C_3}{C_1 + C_2 + C_3} \cdot \frac{1}{j\alpha C_3} = \frac{UC_1}{C_1 + C_2 + C_3} = 57.143 \text{ V}. \tag{55}$$

It is obvious that the voltages on the capacitors C_2, C_3 are the same.

Conclusion

1. The article systematizes a specific class of problems in theoretical electrical engineering. These problems concern the analysis of complex DC electrical circuits containing ideal capacitive and inductive elements. It is argued that traditional methods of analysis for this class of problems are extremely difficult or cannot be applied at all. This is due to the emergence of uncertainties caused by the limiting values of reactances at zero frequency (at DC).

2. To solve the above problems, it is proposed to apply a non-standard analysis, namely, to consider the frequency of the direct current not as zero, but as an infinitely small value. This approach transforms the original problem, making it suitable for solution using classical unified methods of theoretical electrical engineering.

3. The proposed approach, based on the use of non-standard analysis, has demonstrated high efficiency in solving electrical engineering problems on examples of analyzing complex electrical circuits with ideal reactive elements and contour parameters of various orders of infinitesimality.

4. It is advisable to direct further research to identify similar problems in other branches of science and technology, where the use of classical methods is limited due to the presence of boundary processes or uncertain analytical expressions. This will contribute to expanding the scope of application of non-standard analysis as an effective tool for mathematical modeling.

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Электртехникадағы стандартты емес талдау. Идеал реактивті элементтері бар күрделі схемалар

Мақалада теориялық электртехника мәселелеріне стандартты емес талдаудың тұжырымдамалары мен әдістерін қолдану ұсынылған. Стандартты электртехника әдістері жеткілікті тиімді емес, өйткені олар өте күрделі немесе тіпті идеалды индуктивтілік пен сыйымдылықты қамтитын тұрақты ток тізбектерін есептеу үшін жарамсыз. Бұл мәселе тұрақты ток үшін (нөлдік жиілікте) индуктивті кедергі нөлге тең болғандықтан және сыйымдылық кедергісі шексіздікке кететіндіктен туындайды. Нәтижесінде, мұндай электр тізбектерін есептеудің стандартты әдістерін қолданған кезде, типтегі белгісіздік өрнектерін $\frac{0}{0}$ немесе $\frac{\infty}{\infty}$ дәстүрлі математикалық талдау шеңберінде қиындықтар туғызатын мәселелерді шешуге тура келеді. Осы қиындықтарды ескере отырып, классикалық математикалық анализді стандартты емес анализге ауыстыру ұсынылады. Бұл тәсілде тұрақты ток жиілігі нөлдік емес, α шексіз шамасы ретінде қарастырылады. Осы тәсіл индуктивті элементтердің реактивтілігін, сондай-ақ $\frac{1}{\alpha C}$ сыйымдылық элементтерінің реактивтілікті кедергісін анықтауға мүмкіндік береді. Бұл теориялық электртехниканың барлық стандартты әдістерін сәтті қолдануға және белгісіз өрнектермен жұмыс істеу қажеттілігін болдырмауға мүмкіндік береді. Мақалада идеалды индуктивтілігі мен сыйымдылығы бар күрделі тұрақты ток тізбектерін талдаудың нақты мысалдары келтірілген.

Кілт сөздер: тұрақты токтың күрделі тізбектері, стандартты емес талдау, электртехника, идеалды реактивті элемент

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Нестандартный анализ в электротехнике. Сложные схемы с идеальными реактивными элементами

В статье представлено применение концепций и методов нестандартного анализа к проблемам теоретической электротехники. Обосновано, что стандартные методы электротехники недостаточно эффективны, поскольку они чрезмерно сложны или даже непригодны для расчета электрических цепей постоянного тока, содержащих идеальные индуктивности и емкости. Эта проблема возникает из-за того, что для постоянного тока (с нулевой частотой) индуктивное сопротивление равно нулю, а емкостное сопротивление уходит в бесконечность. В результате при использовании стандартных методов расчета таких электрических цепей приходится решать выражения неопределенности типа $\frac{0}{0}$ или $\frac{\infty}{\infty}$, что создает трудности в рамках традиционного математического анализа. Учитывая указанные трудности, предлагается заменить классический математический анализ на нестандартный. При таком подходе частота постоянного тока рассматривается не как нулевая, а как бесконечно малая величина α. Такой подход позволяет определять реактивное сопротивление индуктивных элементов как и реактивное сопротивление емкостных элементов как — $\frac{1}{\alpha C}$. Это позволяет успешно применять все стандартные методы теоретической электротехники и избежать необходимости работы с неопределенными выражениями. В статье приведены конкретные примеры анализа сложных цепей постоянного тока с идеальными индуктивностями и емкостью.

Ключевые слова: сложные цепи постоянного тока, нестандартный анализ, электротехника, идеально-реактивный элемент

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