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Non-standard analysis in electrical engineering. Ideal DC inductive circuits with infinitesimal parameters of different orders

The analysis of DC circuits with ideal inductive elements using standard methods of theoretical electrical engineering is too difficult or even impossible, because when using them, it is often necessary to reveal type $\frac{0}{0}$ uncertainties. In this regard, the article proposes to use not the usual mathematical analysis, but a non-standard one, in which the frequency of the direct current is taken not as zero, but as an infinitesimal number α . In this case, the reactances of the inductive elements will be equal to αL , and it becomes possible to apply all standard methods of theoretical electrical engineering. The article considers examples of the analysis of ideal DC inductive circuits, with particular attention paid to circuits whose calculation requires the use of infinitesimal numbers for inhomogeneous parameters. In such cases, the order of these numbers is determined based on individual considerations, and this is a non-standard task.

Keywords: infinitesimal number, infinitude, hyperreal number, unconventional number, ideal reactive element

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Introduction

When solving various scientific and technical problems, it becomes necessary to reveal type $\frac{0}{0}$ uncertainties, but when using classical methods for this purpose, significant difficulties may arise. Therefore, in such cases, the use of ideas and methods of non-standard analysis is promising. Based on the ideas of non-standard analysis, which consist of and are based on the direct use of infinitesimal numbers, Leibniz and Newton created the foundations for the development of differential and integral calculus. In the scientific works of Cauchy and other scientists, infinitesimal numbers were not used; the mathematical apparatus of differential and integral calculus was created on the basis of numerical and functional sequences and limiting relations of quantities, which ensured its axiomatic rigor, but complicated the solution of a certain range of problems [1–5].

Non-standard analysis began to be widely used in the middle of the last century, when a new axiomatics of mathematical analysis was proposed. This axiomatics is based on the set of hyperreal numbers, which, in addition to standard (real) numbers, also contains non-standard numbers (infinitely small numbers, infinitely large numbers, and their combinations with standard numbers) [6, 7]. Methods of non-standard analysis have been actively developing since the end of the last century to the present and are used in various fields of science and technology [8, 9]. The use of non-standard analysis methods in the identification of internal parameters of electric motors, which in many cases cannot be solved by traditional methods, is promising [10–15]. The use of non-standard analysis allows for effective solutions to some problems in calculating electrical circuits [16–18].

Formulation of the problem

The calculation of DC electrical circuits is usually carried out using unified methods based on Ohm and Kirchhoff's laws. There are certain tasks in this field for which the direct use of unified methods is impossible. Such tasks include, for example, the calculation of DC circuits with ideal reactive elements. The complexity of calculations in such circuits is due to the fact that at constant current the resistance of an ideal inductance tends to zero, and the resistance of an ideal capacitance tends to infinity. Typically, to solve such problems, energy characteristics of inductances and capacitances are used, which significantly complicates the calculation of these circuits, especially for complex circuits. Therefore, it is relevant to use the mathematical apparatus of non-standard analysis, which allows using unified methods for calculating such circuits.

The aim of the article is to identify a class of non-standard electrical engineering problems aimed at analyzing DC electrical circuits that include ideal inductances, and to extend non-standard analysis methods to problems of analyzing electrical circuits with ideal reactive elements. This article considers the non-standard analysis of ideal DC inductive circuits with infinitesimal parameters of different orders.

Research results

Let consider the foundations of the non-standard analysis axiomatics [3, 6, 16–18]. Let denote by R the ordered set of real numbers. The number α is called an infinitesimal number if and only if

$$\forall r \in R(\alpha < r). \tag{1}$$

With infinitesimal numbers, it is possible to perform all algebraic operations (addition, subtraction, multiplication, division, exponentiation, etc.) and apply all theorems (commutativity, associativity, etc.).

Infinitesimal numbers of various orders are used, namely: $\alpha > \alpha^2 > \alpha^3 > \alpha^k$ — infinitesimal numbers of the first, second, third, and *k*-th order. Together with the real numbers $r \in R$, the infinitesimal numbers form an ordered set of hyperreal numbers $*R$.

It is customary to call real numbers $r \in R$ standard or Archimedean numbers in contrast to non-standard (non-Archimedean) numbers $*r \in *R$. Every non-standard number contains a standard part

$$*r = r \pm \alpha, \tag{2}$$

that is,

$$r = st(*r). \tag{3}$$

Thus, an ordinary real number is a standard part of some non-standard number. It is obvious that there can be an infinite number of such numbers. Two standard numbers a and b are equal if and only if

$$a - b = 0. \tag{4}$$

Two non-standard numbers $*a$ and $*b$ are called equivalent, or infinitely close to each other, if and only if

$$*a - *b \approx \alpha. \tag{5}$$

The sign $\approx$ means the equivalence of two non-standard numbers.

For standard numbers m and n, let write the following relations that follow from (1–5):

$$\frac{m\alpha}{n\alpha} = \frac{m}{n}, \frac{m\alpha}{n} = \frac{m}{n}\alpha, m\alpha + n \approx n, m\alpha^k + n \approx n, \sin\alpha \approx \alpha, \cos\alpha \approx 1. \tag{6}$$

Let consider several examples of the use of non-standard numbers in mathematical analysis, namely, that is let determine the derivatives of some functions using non-standard analysis methods. For all the following examples of determining the derivatives of mathematical functions, let introduce the substitution $dx = \alpha$.

Let define the derivative of the function $y = x^n$.

$$\frac{dy}{dx} = \frac{(x+\alpha)^n - x^n}{\alpha} = \frac{x^n + \frac{n!}{1!(n-1)!}x^{n-1}\alpha + \frac{n!}{2!(n-2)!}x^{n-2}\alpha^2 +}{\alpha}\dots$$

$$\dots\frac{+\frac{n!}{2!(n-2)!}x^2\alpha^{n-2} + \frac{n!}{1!(n-1)!}x\alpha^{n-1} + \alpha^n - x^n}{\alpha} = \frac{n!}{1!(n-1)!}x^{n-1} + \frac{n!}{2!(n-2)!}x^{n-2}\alpha + \dots \tag{7}$$

$$\dots + \frac{n!}{2!(n-2)!}x^2\alpha^{n-3} + \frac{n!}{1!(n-1)!}x\alpha^{n-2} + \alpha^{n-1} \approx nx^{n-1}.$$

Let define the derivative of the function $y = \frac{1}{x^n}$.

$$\frac{dy}{dx}=\frac{\frac{1}{(x+\alpha)^n}-\frac{1}{x^n}}{\alpha}=\frac{\frac{1}{x^n+\frac{n!}{1!(n-1)!}x^{n-1}\alpha+\ldots+\frac{n!}{1!(n-1)!}x\alpha^{n-1}+\alpha^n}-\frac{1}{x^n}}{\alpha}=$$

$$=\frac{\dfrac{x^n-x^n-\frac{n!}{1!(n-1)!}x^{n-1}\alpha-\ldots-\frac{n!}{1!(n-1)!}x\alpha^{n-1}-\alpha^n}{x^{2n}+\frac{n!}{1!(n-1)!}x^{2n-1}\alpha+\ldots+\frac{n!}{1!(n-1)!}x^{n+1}\alpha^{n-1}+x^n\alpha^n}}{\alpha}= \tag{8}$$

$$=\frac{-\frac{n!}{1!(n-1)!}x^{n-1}-\ldots-\frac{n!}{1!(n-1)!}x\alpha^{n-2}-\alpha^{n-1}}{x^{2n}+\frac{n!}{1!(n-1)!}x^{2n-1}\alpha+\ldots+\frac{n!}{1!(n-1)!}x^{n+1}\alpha^{n-1}+x^n\alpha^n}\approx\frac{-\frac{n!}{1!(n-1)!}x^{n-1}}{x^{2n}}=-\frac{n}{x^{n+1}}.$$

Let define the derivative of the function $y=\sqrt{x}$.

$$\frac{dy}{dx}=\frac{\sqrt{x+\alpha}-\sqrt{x}}{\alpha}=\frac{\left(\sqrt{x+\alpha}-\sqrt{x}\right)\left(\sqrt{x+\alpha}+\sqrt{x}\right)}{\alpha\left(\sqrt{x+\alpha}+\sqrt{x}\right)}=\frac{x+\alpha-x}{\alpha\left(\sqrt{x+\alpha}+\sqrt{x}\right)}=\frac{1}{\left(\sqrt{x+\alpha}+\sqrt{x}\right)}\approx\frac{1}{2\sqrt{x}}. \tag{9}$$

Let define the derivative of the function $y=\operatorname{tg}x$.

$$\frac{dy}{dx}=\frac{\frac{\sin(x+\alpha)}{\cos(x+\alpha)}-\frac{\sin x}{\cos x}}{\alpha}=\frac{\frac{\sin x\cos\alpha+\sin\alpha\cos x}{\cos x\cos\alpha-\sin x\sin\alpha}-\frac{\sin x}{\cos x}}{\alpha}=$$

$$=\frac{\cos x\sin x\cos\alpha+\sin\alpha\cos^2 x-\sin x\cos x\cos\alpha+\sin^2 x\sin\alpha}{\alpha\cos^2 x\cos\alpha-\alpha\cos x\sin x\sin\alpha}\approx \tag{10}$$

$$\approx\frac{\alpha\left(\cos^2 x+\sin^2 x\right)}{\alpha\cos^2 x-\alpha^2\cos x\cdot\sin x}\approx\frac{\cos^2 x+\sin^2 x}{\cos^2 x-\alpha\cos x\cdot\sin x}\approx\frac{1}{\cos^2 x}.$$

Let define the derivative of the function $y=\operatorname{ctg}x$.

$$\frac{dy}{dx}=\frac{\frac{\cos(x+\alpha)}{\sin(x+\alpha)}-\frac{\cos x}{\sin x}}{\alpha}=\frac{\frac{\cos x\cos\alpha-\sin x\sin\alpha}{\sin x\cos\alpha+\sin\alpha\cos x}-\frac{\cos x}{\sin x}}{\alpha}=$$

$$=\frac{\sin x\cos x\cos\alpha-\sin^2 x\sin\alpha-\cos x\sin x\cos\alpha-\sin\alpha\cos^2 x}{\alpha\sin^2 x\cos\alpha+\alpha\sin\alpha\cos x\sin x}\approx \tag{11}$$

$$\approx\frac{-\alpha\left(\sin^2 x+\cos^2 x\right)}{\alpha\sin^2 x+\alpha^2\cos x\sin x}\approx\frac{-\left(\sin^2 x+\cos^2 x\right)}{\sin^2 x+\alpha\cos x\sin x}\approx\frac{-1}{\sin^2 x}.$$

Let define the derivative of the function $y=\sec x$. Because $\sec x=\frac{1}{\cos x}$, that

$$\frac{dy}{dx}=\frac{\frac{1}{\cos(x+\alpha)}-\frac{1}{\cos x}}{\alpha}=\frac{\frac{1}{\cos x\cos\alpha-\sin x\sin\alpha}-\frac{1}{\cos x}}{\alpha}= \tag{12}$$

$$=\frac{\cos x-\cos x\cos\alpha+\sin x\sin\alpha}{\alpha\cos^2 x\cos\alpha-\alpha\cos x\sin x\sin\alpha}\approx\frac{\alpha\sin x}{\alpha\cos^2 x-\alpha^2\cos x\sin x}\approx\frac{\sin x}{\cos^2 x-\alpha\cos x\cdot\sin x}\approx\frac{\sin x}{\cos^2 x}.$$

Let define the derivative of the function $y=\operatorname{cosec}(x)$. Because $\operatorname{cosec}(x)=\frac{1}{\sin x}$, that

$$\frac{dy}{dx}=\frac{\frac{1}{\sin(x+\alpha)}-\frac{1}{\sin x}}{\alpha}=\frac{\frac{1}{\sin x\cos\alpha+\sin\alpha\cos x}-\frac{1}{\sin x}}{\alpha}=$$
$$=\frac{\sin x-\sin x\cos\alpha-\sin\alpha\cos x}{\alpha\sin^2 x\cos\alpha+\alpha\sin\alpha\cos x\sin x}\approx\frac{-\alpha\cos x}{\alpha\sin^2 x+\alpha^2\cos x\sin x}\approx-\frac{\cos x}{\sin^2 x}. \tag{13}$$

Let define the derivative of the function $y=e^x$. Because $e=\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^n$, then within the framework of non-standard analysis we can write $e\approx(1+\alpha)^{\frac{1}{\alpha}}$, $e^x\approx(1+\alpha)^{\frac{x}{\alpha}}$.

$$\frac{dy}{dx}\approx\frac{(1+\alpha)^{\frac{x+\alpha}{\alpha}}-(1+\alpha)^{\frac{x}{\alpha}}}{\alpha}=\frac{(1+\alpha)^{\frac{x}{\alpha}}(1+\alpha)^{\frac{\alpha}{\alpha}}-(1+\alpha)^{\frac{x}{\alpha}}}{\alpha}=\frac{(1+\alpha)^{\frac{x}{\alpha}}(1+\alpha)^{\frac{\alpha}{\alpha}}-(1+\alpha)^{\frac{x}{\alpha}}}{\alpha}=$$
$$=\frac{(1+\alpha)^{\frac{x}{\alpha}}\left[(1+\alpha)^{\frac{\alpha}{\alpha}}-1\right]}{\alpha}==\frac{(1+\alpha)^{\frac{x}{\alpha}}[1+\alpha-1]}{\alpha}=(1+\alpha)^{\frac{x}{\alpha}}\approx e^x. \tag{14}$$

Not only the set of real numbers, but also the set of complex numbers can have the same structure. Based on this, taking into account (6), we can write:

$$m\alpha+jn\approx jn, m+jn\alpha\approx m. \tag{15}$$

Next, we will consider how non-standard analysis methods can be used to analyze DC circuits with ideal inductive elements. Before proceeding to the application of the above expressions to solve applied problems, it should be noted that there are no general rules for choosing a parameter that should be equated to an infinitely small or infinitely large number. This choice is made by the researcher depending on the context of the specific task. It should be noted that in the case of the need to replace several heterogeneous parameters in one problem with infinitesimal numbers, determining the relationships between these numbers is in most cases very difficult and sometimes requires additional research.

Next, let consider examples of solving problems using inhomogeneous infinitesimal parameters. Since a DC circuit can be considered as a sinusoidal AC circuit, with an alternating current of zero frequency, the symbolic method can be used to solve such problems, provided that $\omega=\alpha$. Taking $\omega=\alpha$, for the complex resistance of the inductance we can write

$$\underline{Z}_L\approx j\alpha L. \tag{16}$$

Consider the first example, in a DC circuit (Fig. 1), the task is to determine the currents in all branches of the circuit.

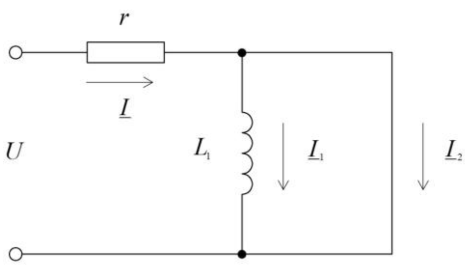

Figure 1. Circuit with shorted ideal inductance

In the circuit shown in Fig. 1, the inductance L_1 is short-circuited. The question arises as to how best to represent a branch that is switched on in parallel with L_1. If this branch is represented as an ideal resistor R with zero resistance, then two different infinitesimal numbers $\omega\approx\alpha_1$ and $R\approx\alpha_2$ will appear simultaneously in the problem. At the same time, the ratio between α_1 and α_2 is unknown to us, and it is difficult to even estimate this ratio. Therefore, it is appropriate to represent this branch as an ideal inductance $\underline{Z}_2$, in which $L_2\approx\alpha_2$. In this case, the total complex resistance of this branch can be written as

$$\underline{Z} \approx j\alpha_1\alpha_2 . \tag{17}$$

The product of two infinitesimal numbers is an infinitesimal number of higher order than each of the factors, so one of the inequalities holds

$$j\alpha_1\alpha_2 \le jm\alpha_1^2 , \tag{18}$$

or

$$j\alpha_1\alpha_2 \le jm\alpha_2^2 . \tag{19}$$

Let's solve the problem for the first inequality (obviously, the result will be similar for the second). The total complex resistance of the circuit is given by the expression

$$\underline{Z}_{вх} \approx r + \frac{\left(j\alpha_1 L_1\right)\left(jm\alpha_1^2\right)}{j\alpha_1 L_1 + jm\alpha_1^2} = r + \frac{j^2 m\alpha_1^3 L_1}{j\alpha_1\left(L_1 + m\alpha_1\right)} = r + jm\alpha_1^2 \frac{L_1}{\left(L_1 + m\alpha_1\right)} , \tag{20}$$

and according to (15)

$$\underline{Z}_{вх} \approx r . \tag{21}$$

Hence $\underline{I} = \frac{U}{r}$, and the voltage on the parallel branches is

$$\underline{U}_L = jm\alpha_1^2 \frac{L_1}{\left(L_1 + m\alpha_1\right)} \underline{I} = \frac{U}{r} jm\alpha_1^2 \frac{L_1}{\left(L_1 + m\alpha_1\right)} , \tag{22}$$

and after transformations, we get

$$\underline{U}_L = \frac{U}{r} jm\alpha_1^2 \frac{L_1}{\left(L_1 + m\alpha_1\right)} \approx \frac{U}{r} jm\alpha_1^2 \frac{L_1}{L_1} = \frac{U}{r} jm\alpha_1^2 . \tag{23}$$

Then the current in the branch with inductance is determined by the expression

$$\underline{I}_1 = \frac{\underline{U}_L}{j\alpha_1 L_1} = \frac{U}{r} \frac{jm\alpha_1^2}{j\alpha_1 L_1} = \frac{U}{r} \frac{m}{L_1} \alpha_1 \approx 0 , \tag{24}$$

and in the parallel branch

$$\underline{I}_2 = \frac{\underline{U}_L}{jm\alpha_1^2} = \frac{U}{r} \frac{jm\alpha_1^2}{jm\alpha_1^2} = \frac{U}{r} . \tag{25}$$

More interesting is the case when, in a circuit with ideal inductances, the active resistances of the coils are infinitesimal numbers of different orders. Consider the second example, the DC circuit, which is shown on the Figure 2.

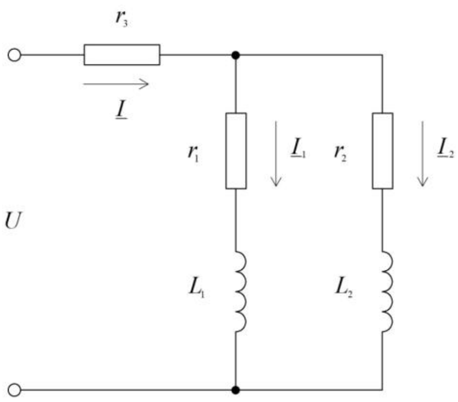

Figure 2. Circuit with ideal inductances in which the active resistances of the coils are infinitesimal numbers of different orders

The electric circuit shown in Figure 2 will be considered a circuit with ideal inductances if the resistances r_1, r_2 are infinitely small numbers, i.e. $r_1 \approx \alpha_1$, $r_2 \approx \alpha_2$. This problem, it is necessary to consider three different infinitesimal numbers $\omega \approx \alpha$, $r_1 \approx \alpha_1$, $r_2 \approx \alpha_2$, the order ratio of which is unknown in advance.

Let consider possible variants of the relations between these different infinitesimal numbers. In this case, it makes sense to assume that the resistances r_1, r_2, are infinitesimal numbers of the same order. There are a total of three subsequent variants of the above relationships.

Variant 1. The order of the direct current frequency ω is equal to the order of the resistances r_1, r_2, i.e.

$$\alpha \approx \alpha_1 \approx \alpha_2 . \tag{26}$$

Variant 2. The order of the direct current frequency ω is greater than the order of the resistances r_1, r_2, i.e.

$$\alpha < \left(\alpha_1 \approx \alpha_2\right). \tag{27}$$

Variant 3. The order of the direct current frequency ω is less than the order of the resistances r_1, r_2, i.e.

$$\alpha > \left(\alpha_1 \approx \alpha_2\right). \tag{28}$$

Let determine the currents in the branches of the circuit for each of the above variants. Let consider the first variant. In general form, we can write

$$\alpha_1 \approx m\alpha ,\ \alpha_2 \approx n\alpha , \tag{29}$$

where m, n — arbitrary standard numbers (in particular 1).

Then for the impedances of the circuit branches (Fig. 2) we can write

$$\underline{Z}_1 = r_1 + j\omega L_1 = \alpha_1 + j\alpha L_1 = m\alpha + j\alpha L_1 = \alpha\left(m + jL_1\right), \tag{30}$$

$$\underline{Z}_2 = r_2 + j\omega L_2 = \alpha_2 + j\alpha L_2 = n\alpha + j\alpha L_2 = \alpha\left(n + jL_2\right), \tag{31}$$

$$\underline{Z}_3 = r_3 . \tag{32}$$

The impedance of the entire circuit (Fig. 2) is given by the expression

$$\underline{Z}_{вх} = \underline{Z}_3 + \frac{\underline{Z}_1\underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2} = r_3 + \frac{\alpha\left(m + jL_1\right)\alpha\left(n + jL_2\right)}{\alpha\left(m + jL_1\right) + \alpha\left(n + jL_2\right)} = r_3 + \frac{\alpha\left(m + jL_1\right)\left(n + jL_2\right)}{m + n + j\left(L_1 + L_2\right)} \approx r_3. \tag{33}$$

Hence $\underline{I} = \dfrac{U}{r_3}$, and the voltage on the parallel branches $\underline{Z}_1$ and $\underline{Z}_2$

$$\underline{U}_Z = \underline{I}\,\frac{\alpha\left(m + jL_1\right)\left(n + jL_2\right)}{m + n + j\left(L_1 + L_2\right)} = \frac{U}{r}\,\frac{\alpha\left(m + jL_1\right)\left(n + jL_2\right)}{m + n + j\left(L_1 + L_2\right)}, \tag{34}$$

and the currents in the branches

$$\underline{I}_1 = \frac{\underline{U}_Z}{\underline{Z}_1} = \frac{\dfrac{U}{r}\,\dfrac{\alpha\left(m + jL_1\right)\left(n + jL_2\right)}{m + n + j\left(L_1 + L_2\right)}}{\alpha\left(m + jL_1\right)} = \frac{U\left(n + jL_2\right)}{r\left[m + n + j\left(L_1 + L_2\right)\right]}, \tag{35}$$

$$\underline{I}_2 = \frac{\underline{U}_Z}{\underline{Z}_2} = \frac{\dfrac{U}{r}\,\dfrac{\alpha\left(m + jL_1\right)\left(n + jL_2\right)}{m + n + j\left(L_1 + L_2\right)}}{\alpha\left(n + jL_2\right)} = \frac{U\left(m + jL_1\right)}{r\left[m + n + j\left(L_1 + L_2\right)\right]}. \tag{36}$$

As follows from expressions (35) and (36), in the first variant, despite the constant voltage, the currents in the inductances are complex quantities.

Let consider the second variant. In general form, we can write

$$\alpha \approx m\alpha_1^k ,\ \alpha_2 \approx n\alpha_1 , \tag{37}$$

where m, n — arbitrary standard numbers (in particular 1).

Then for the impedances of the circuit branches (Fig. 2), we can write

$$\underline{Z}_1 = r_1 + j\omega L_1 = \alpha_1 + j\alpha L_1 = \alpha_1 + jm\alpha_1^k L_1, \tag{38}$$

$$\underline{Z}_2 = r_2 + j\omega L_2 = \alpha_2 + j\alpha L_2 = n\alpha_1 + jm\alpha_1^k L_2, \tag{39}$$

$$\underline{Z}_3 = r_3. \tag{40}$$

The impedance of the entire circuit (Fig. 2) is given by the expression

$$\underline{Z}_{вх} = \underline{Z}_3 + \frac{\underline{Z}_1\underline{Z}_2}{\underline{Z}_1+\underline{Z}_2} = r_3 + \frac{\left(\alpha_1 + jm\alpha_1^k L_1\right)\left(n\alpha_1 + jm\alpha_1^k L_2\right)}{\alpha_1 + jm\alpha_1^k L_1 + n\alpha_1 + jm\alpha_1^k L_2} = r_3 + \frac{\left(\alpha_1 + jm\alpha_1^k L_1\right)\left(n + jm\alpha_1^{k-1} L_2\right)}{1 + jm\alpha_1^{k-1} L_1 + n + jm\alpha_1^{k-1} L_2} =$$
$$= r_3 + \frac{\left(\alpha_1 + jm\alpha_1^k L_1\right)\left(n + jm\alpha_1^{k-1} L_2\right)}{n+1+jm\alpha_1^{k-1}\left(L_1+L_2\right)} \approx r_3. \tag{41}$$

Hence $\underline{I} = \frac{U}{r_3}$, and the voltage on the parallel branches $\underline{Z}_1$ and $\underline{Z}_2$

$$\underline{U}_Z = \underline{I}\,\frac{\left(\alpha_1 + jm\alpha_1^k L_1\right)\left(n + jm\alpha_1^{k-1} L_2\right)}{n+1+jm\alpha_1^{k-1}\left(L_1+L_2\right)} = \frac{U}{r}\,\frac{\left(\alpha_1 + jm\alpha_1^k L_1\right)\left(n + jm\alpha_1^{k-1} L_2\right)}{n+1+jm\alpha_1^{k-1}\left(L_1+L_2\right)}, \tag{42}$$

and the currents in the branches

$$\underline{I}_1 = \frac{\underline{U}_Z}{\underline{Z}_1} = \frac{\dfrac{U}{r}\,\dfrac{\left(\alpha_1 + jm\alpha_1^k L_1\right)\left(n + jm\alpha_1^{k-1} L_2\right)}{n+1+jm\alpha_1^{k-1}\left(L_1+L_2\right)}}{\left(\alpha_1 + jm\alpha_1^k L_1\right)} = \frac{U\left(n + jm\alpha_1^{k-1} L_2\right)}{r\left[n+1+jm\alpha_1^{k-1}\left(L_1+L_2\right)\right]} \approx \frac{Un}{r(n+1)}, \tag{43}$$

$$\underline{I}_2 = \frac{\underline{U}_Z}{\underline{Z}_2} = \frac{\dfrac{U}{r}\,\dfrac{\left(\alpha_1 + jm\alpha_1^k L_1\right)\left(n + jm\alpha_1^{k-1} L_2\right)}{n+1+jm\alpha_1^{k-1}\left(L_1+L_2\right)}}{n\alpha_1 + jm\alpha_1^k L_2} = \frac{U}{r}\,\frac{\left(1 + jm\alpha_1^{k-1} L_1\right)}{n+1+jm\alpha_1^{k-1}\left(L_1+L_2\right)} \approx \frac{U}{r(n+1)}. \tag{44}$$

As follows from expressions (43) and (44), in the second variant, the currents in the inductances do not depend on the values of the inductances themselves.

Let consider the third variant. In general form, we can write

$$\alpha_1 \approx m\alpha^k,\ \alpha_2 \approx n\alpha^k, \tag{45}$$

where m, n — arbitrary standard numbers (in particular 1).

Then for the impedances of the circuit branches (Fig. 2) we can write

$$\underline{Z}_1 = r_1 + j\omega L_1 = \alpha_1 + j\alpha L_1 = m\alpha^k + j\alpha L_1 = \alpha\left(m\alpha^{k-1} + jL_1\right), \tag{46}$$

$$\underline{Z}_2 = r_2 + j\omega L_2 = \alpha_2 + j\alpha L_2 = n\alpha^k + j\alpha L_2 = \alpha\left(n\alpha^{k-1} + jL_2\right), \tag{47}$$

$$\underline{Z}_3 = r_3. \tag{48}$$

The impedance of the entire circuit (Fig. 2) is given by the expression

$$\underline{Z}_{вх} = \underline{Z}_3 + \frac{\underline{Z}_1\underline{Z}_2}{\underline{Z}_1+\underline{Z}_2} = r_3 + \frac{\alpha\left(m\alpha^{k-1} + jL_1\right)\alpha\left(n\alpha^{k-1} + jL_2\right)}{\alpha\left(m\alpha^{k-1} + jL_1\right) + \alpha\left(n\alpha^{k-1} + jL_2\right)} = r_3 + \frac{\alpha\left(m\alpha^{k-1} + jL_1\right)\left(n\alpha^{k-1} + jL_2\right)}{(m+n)\alpha^{k-1} + j\left(L_1+L_2\right)} \approx r_3. \tag{49}$$

Hence $\underline{I} = \frac{U}{r_3}$, and the voltage on the parallel branches $\underline{Z}_1$ and $\underline{Z}_2$

$$\underline{U}_Z = \underline{I}\,\frac{\alpha\left(m\alpha^{k-1} + jL_1\right)\left(n\alpha^{k-1} + jL_2\right)}{(m+n)\alpha^{k-1} + j\left(L_1+L_2\right)} = \frac{U}{r}\,\frac{\alpha\left(m\alpha^{k-1} + jL_1\right)\left(n\alpha^{k-1} + jL_2\right)}{(m+n)\alpha^{k-1} + j\left(L_1+L_2\right)}, \tag{50}$$

and the currents in the branches

$$\underline{I}_1 = \frac{\underline{U}_Z}{\underline{Z}_1} = \frac{\dfrac{U}{r}\,\dfrac{\alpha\left(m\alpha^{k-1} + jL_1\right)\left(n\alpha^{k-1} + jL_2\right)}{(m+n)\alpha^{k-1} + j\left(L_1 + L_2\right)}}{\alpha\left(m\alpha^{k-1} + jL_1\right)} = \frac{U\left(n\alpha^{k-1} + jL_2\right)}{r\left[(m+n)\alpha^{k-1} + j\left(L_1 + L_2\right)\right]} \approx \frac{UL_2}{r\left(L_1 + L_2\right)}, \qquad (51)$$

$$\underline{I}_2 = \frac{\underline{U}_Z}{\underline{Z}_2} = \frac{\dfrac{U}{r}\,\dfrac{\alpha\left(m\alpha^{k-1} + jL_1\right)\left(n\alpha^{k-1} + jL_2\right)}{(m+n)\alpha^{k-1} + j\left(L_1 + L_2\right)}}{\alpha\left(n\alpha^{k-1} + jL_2\right)} = \frac{U\left(m\alpha^{k-1} + jL_1\right)}{r\left[(m+n)\alpha^{k-1} + j\left(L_1 + L_2\right)\right]} \approx \frac{UL_1}{r\left(L_1 + L_2\right)}. \qquad (52)$$

As follows from expressions (43) and (44), in the third variant, the currents in the inductances depend only on the values of the inductances themselves. It is this variant that conceptually corresponds to the applied problems of the theory of electrical circuits.

Conclusions

1. The authors first identified the class of non-standard electrical engineering problems aimed at the analysis of DC electrical circuits that include ideal inductances. It has been proven that solving a selected class of problems using standard methods of theoretical electrical engineering is very difficult, or sometimes impossible.

2. In order to solve the identified problems, it is proposed to use non-standard analysis methods for analyzing DC electric circuits with ideal inductances. The advantages of implementing such an approach are confirmed by examples of calculations of electrical circuits with inductances and inhomogeneous infinitesimal parameters.

3. To expand the scope of use of non-standard analysis methods, it is recommended to outline similar tasks in those branches of science and technology in which limit transitions and differential calculus are used, and in which the solution of the corresponding tasks by standard methods is significantly limited or impossible.

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Электртехникадағы стандартты емес талдау. Әр түрлі ретті шексіз кіші параметрлері бар тұрақты токтың идеалды индуктивті тізбектері

Теориялық электртехниканың стандартты әдістерін қолдана отырып, идеалды индуктивті элементтері бар тұрақты ток DC тізбектерін талдау өте күрделі немесе тіпті мүмкін емес болуы мүмкін, өйткені оларды пайдалану кезінде көбінесе $\frac{0}{0}$ типтік белгісіздіктерді анықтау қажет. Осыған байланысты мақалада тұрақты ток жиілігі нөлге тең емес, α шексіз аз сан ретінде қабылданатын стандартты емес талдауды қолдану ұсынылған. Бұл жағдайда индуктивті элементтердің реактивтері αL тең болады және теориялық электртехниканың барлық стандартты әдістерін қолдануға болады. Мақалада тұрақты токтың идеалды индуктивті тізбектерін DC талдау мысалдары қарастырылған, олардың есебі гетерогенді параметрлер үшін шексіз аз сандарды қолдануды қажет ететін тізбектерге ерекше назар аударылған. Мұндай жағдайларда бұл сандардың орналасу реті жеке ойлар негізінде анықталады және бұл стандартты емес тапсырма.

Кілт сөздер: шексіз аз сан, шексіздік, гиперреал сандар, дәстүрлі емес сандар, идеалды реактивті элемент

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Нестандартный анализ в электротехнике. Идеальные индуктивные цепи постоянного тока с бесконечно малыми параметрами разного порядка

Анализ цепей DC постоянного тока с идеальными индуктивными элементами с использованием стандартных методов теоретической электротехники слишком сложен или даже невозможен, поскольку при их использовании часто возникает необходимость выявления неопределенностей типа $\frac{0}{0}$. В связи с этим в статье предлагается использовать не обычный математический анализ, а нестандартный, при котором частота постоянного тока берется не за ноль, а за бесконечно малое число α. В этом случае реактивные сопротивления индуктивных элементов будут равны αL, и становится возможным применять все стандартные методы теоретической электротехники. В статье рассматриваются примеры анализа идеальных индуктивных цепей DC постоянного тока, при этом особое внимание уделяется цепям, расчет которых требует использования бесконечно малых чисел для неоднородных параметров. В таких случаях порядок расположения этих чисел определяется исходя из индивидуальных соображений, и это нестандартная задача.

Ключевые слова: бесконечно малое число, бесконечность, гиперреальные числа, нетрадиционное число, идеальный реактивный элемент

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