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## Influence of a Constant Magnetic Field and a High-Frequency Electric Field on Plasma

A new method of determining the expression of the averaged high-frequency pressure force (Miller force) based on the solution of the kinetic equation for the electron distribution function and the method of successive approximations (separation of slow motions and fast oscillations) is proposed for the case when a high-frequency electric field and a stationary magnetic field acting simultaneously on weakly inhomogeneous plasma. Moreover, widely recognized methods of theoretical and mathematical physics, such as averaging over the period of oscillation of the electric field and integration over the trajectory, have been applied. Collisions between electrons and stationary ions have been considered. The electric field amplitude is considered a slowly varying function of time and coordinates. The obtained expression allows us to estimate the influence of collisions of plasma particles on the Miller force, and under limiting conditions it coincides with the known expressions for the high-frequency pressure force derived from the equation of plasma electrons motion in high-frequency fields. The calculations neglect the contribution of the magnetic component of the electromagnetic field, which is applicable to longitudinal electric fields. The results obtained in this article are primarily of theoretical interest and reveal the interaction of weakly inhomogeneous plasma with a high-frequency electric field. They can also be used in constructing the kinetic theory of inhomogeneous plasma in high frequency electromagnetic fields.

*Keywords:* weakly inhomogeneous plasma, plasma electrons, kinetic equation, averaged force, electric field, particle collisions, fixed ions, high frequency

### Introduction

It is known that in high-frequency electromagnetic fields, particles (specifically, electrons) experience not only the generalized Lorentz force  $\vec{F}_0$  but also an additional force  $\vec{f}_M$  determined by the high-frequency quasi-potential  $U_e$ , i.e.,  $\vec{f}_M = -n_e \text{grad}_r U_e$  where  $n_e$  is the electron concentration. As a result of its action, the plasma electrons tend to move to the field minimum. The physics of such a phenomenon can be explained as follows. The electromagnetic field, causing high-frequency oscillations of electrons with velocity  $\vec{v}_e$  creates as if an additional high-frequency pressure  $P_{eq} \sim n_e m_e v_e^2$  (or  $P_{eq} \sim n_e U_e$ , where  $m_e$  is the electron mass), as a result of which the plasma electrons are displaced from the areas occupied by the field. In oscillatory motion, the force  $\vec{f}_M = -\text{grad}_r P_{eq}$  is directed against the displacement of electrons, so when an electron is displaced to the right, a return force of greater magnitude acts on it compared to when it is displaced to the left. The averaged force  $\vec{f}_M$ , so called the high-frequency pressure force, or sometimes the quasi-potential Miller force, do not depend on the particle charge sign. An expression for the studied force, based on the equation of motion of electrons, was defined in the works of a number of authors [1–4], devoted to the acceleration of plasma particles and the confinement of high-temperature plasma by high-frequency electromagnetic fields. In this article, the expression for the Miller force is derived from the kinetic equation for the electron distribution function, taking into account electron-ion collisions and the influence of longitudinal, high-frequency, and inhomogeneous electric fields and stationary magnetic fields on weakly inhomogeneous magnetically active plasma. This is done using the method of successive approximations (separation of slow motions and fast oscillations).

In this regard, this article addresses the problem of the impact of high-frequency electric and constant magnetic fields on weakly inhomogeneous plasma. In particular, the contributions of external fields to the kinetics of weakly inhomogeneous plasma in the approximation of pair collisions between particles are evaluated. Expressions for the collision integrals of electrons with electrons and electrons with stationary ions, as well as for the force of high-frequency pressure, are also determined, taking into account the

presence of a high-frequency longitudinal electric field and a constant magnetic field. Despite the increased interest in research in this direction, the question of the mechanisms of plasma behavior in external fields is far from fully studied and remains open, which makes the topic of the article relevant.

### *Materials and Methods*

Motion of charged particles in electric and magnetic fields based on classical physics concepts, has been studied in papers [5–8]. These concepts not only remain valid when analyzing the motion of charged particles under the influence of macroscopic external fields but also form the foundation required for understanding processes of particle interaction in plasma — processes involving the microscopic fields of single particles. When analytically investigating various properties of plasma, one of the common methods is the kinetic equations approach. Although the Landau, Klimontovich, Lenard-Balescu-Silin, etc. equations have long been known in kinetic theory, the interest in this area of plasma physics remains unabated. In addition, widely recognized methods of theoretical and mathematical physics have been applied in plasma research, such as averaging over the period of oscillation of the electric field and integration over trajectory.

### *Literature review*

The external high-frequency pressure force from the equation of motion is considered in the literature [1–4] in the case of a plasma without an external magnetic field and homogeneous. In the literature [5–10], one can become familiar with the interactions between plasma and external fields, kinetic processes, and electrophysical laws. Additionally, the results of recent years, which have been added to the kinetic theory of ionized plasma, are analyzed and discussed in references [11–18].

### *Main part. Experimental*

Assume that the plasma, except for the high-frequency field

$$\vec{E} = \vec{E}_0(\varepsilon\vec{r}, \varepsilon t) \sin \omega_0 t \quad (1)$$

is influenced by an external constant magnetic field  $\vec{B}_0$ , oriented along the axis  $z$ . The kinetic equation for the electron distribution function  $F_e$ , considering the collisions of electrons with ions and effecting external fields, is written in the form of [9–12]

$$\begin{aligned} \frac{\partial F_e}{\partial t} + \vec{v} \frac{\partial F_e}{\partial \vec{r}} - \vec{F}_o \frac{\partial F_e}{\partial \vec{p}} = St_{ei} \{F_e\}, \\ \vec{F}_o = e \left( \vec{E} + \frac{1}{c} [\vec{v} \vec{B}_0] \right), \end{aligned} \quad (2)$$

where  $\vec{v} = \vec{p} / m_e$  is the electron velocity. The field amplitude  $\vec{E}_0$  is considered a slowly varying function of time  $t$  and as per coordinates  $\vec{r}$ . The parameter  $\varepsilon$  characterizing the slowness of the amplitude change, fulfills the condition  $\varepsilon = (V_T / \omega_0 L) \ll 1$ , where  $V_T$  is the electron thermal velocity,  $\omega_0$  is the frequency, and  $L$  is typical value of the change  $F_e$ . When referring to high field frequencies, we assume that during the period of field oscillation, the electron travels a distance much shorter than its free path length, and thus can be considered to move in an almost homogeneous field [13–16]. Here  $l_{\omega_0} = V_T \omega_0^{-1}$  and  $l = V_T \nu_{ei}^{-1}$ ;  $\nu_{ei}$  is the frequency of electron-ion collisions. We will assume that the plasma is weakly inhomogeneous, i.e., the functions  $F_e$  and  $\vec{E}$  do not change much at a distance of the Debye radius  $r_d$  or the condition  $(L / \nu_0 T) \gg 1$  is fulfilled, where  $\nu_0$  is the average particle velocity.

As known, the collision integral characterizes the change in the particle distribution function due to collisions between them. It has dimensionality  $F / t$ , where is the  $F$  distribution function. The characteristic time  $t$  of change  $F$  due to time-of-collisions between particles, i.e.  $t \sim \tau$ . Furthermore, the collision integral becomes zero when  $F$  coincides with Maxwell distribution function.

Therefore, in (2), the collision integral is determined using the following approximate expression ( $\tau$ -approximation):  $St_{ei} \{F_e\} = -\nu_{ei} (F_e - \langle F_e^0 \rangle)$ , here the relation is taken onto account  $\nu_{ei} \sim 1 / \tau_{ei}$ . Substituting (2), we obtain:

$$\left\{ \begin{array}{l} \hat{L}_2 F_e^0 = v_{ei} < F_e^{0(0)} >; \\ \hat{L}_2 F_e^1 = -\mathbf{v} \frac{\partial F_e^0}{\partial \varepsilon \vec{r}}, \end{array} \right. \quad (3)$$

$$\left\{ \begin{array}{l} \hat{L}_2 F_e^0 = v_{ei} < F_e^{0(0)} >; \\ \hat{L}_2 F_e^1 = -\mathbf{v} \frac{\partial F_e^0}{\partial \varepsilon \vec{r}}, \end{array} \right. \quad (4)$$

where

$$\hat{L}_2 = \frac{\partial}{\partial t} - \vec{F}_0 \frac{\partial}{\partial \vec{p}} + v_{ei}.$$

In the presence of  $\vec{B}_0$  a small parameter  $\varepsilon$  the relation can be determined by  $\varepsilon = (V_T / \Delta \omega L) \ll 1$ , where  $\Delta \omega = (\omega_0 - \omega_H)$  is offset frequency;  $\omega_H = eB_0 / m_e c$  is the electron cyclotron frequency,  $c$  is the light velocity. Under the condition  $(\omega_H / \omega_0) < \varepsilon$  we have  $\Delta \omega \approx \omega_0$ . Since  $\vec{p}(p_x, p_y, p_z)$ ,  $\vec{r}(r_x, r_y, r_z)$  and  $\vec{E}(E_{0x}, E_{0y}, E_{0z})$  the equations (3) and (4) can be rewritten as:

$$\hat{L}_3 F_e^0 = v_{ei} < F_e^{0(0)} >; \quad (3A)$$

$$\hat{L}_3 F_e^1 = -S_1, \quad (4A)$$

where, for simplicity, the following notations were used:

$$\hat{L}_3 = \frac{\partial}{\partial t} - M_1 \frac{\partial}{\partial p_x} - M_2 \frac{\partial}{\partial p_y} - M_3 \frac{\partial}{\partial p_z};$$

$$S_1 = v_x \frac{\partial F_e^0}{\partial \varepsilon r_x} - v_y \frac{\partial F_e^0}{\partial \varepsilon r_y} - v_z \frac{\partial F_e^0}{\partial \varepsilon r_z};$$

$$M_1 = eE_{0x} \sin \omega_0 t + p_y \omega_H;$$

$$M_2 = eE_{0y} \sin \omega_0 t - p_x \omega_H;$$

$$M_3 = eE_{0z} \sin \omega_0 t.$$

The approximate solutions to equations (3A) and (4A) can be expressed as:

$$F_e^0 = F_e^{0(0)} + F_e^{0(1)}; \quad (5)$$

$$F_e^1 = F_e^{1(0)} + F_e^{1(1)}, \quad (6)$$

where the first terms on the right-hand sides of (5) and (6) are solutions to the homogeneous equations  $\hat{L}_3 F_e^{0(0)} = 0$  and  $\hat{L}_3 F_e^{1(0)} = 0$  respectively, expressed by the equality:

$$\begin{aligned} F_e^{0(0)} = F_e^{1(0)} = n_e (2\pi m_e k T_e)^{-3/2} \exp\{-(2m_e k T_e)^{-1} [(p_x - \frac{e\omega_0 E_{0x}}{\omega_0^2 - \omega_H^2} \cos \omega_0 t)^2 + \\ + (p_y - \frac{e\omega_0 E_{0y}}{\omega_0^2 - \omega_H^2} \cos \omega_0 t)^2 + (p_z - \frac{eE_{0z}}{\omega_0} \cos \omega_0 t)^2 + (\frac{e\omega_H}{\omega_0^2 - \omega_H^2})^2 \times \\ \times (E_{0x}^2 + E_{0y}^2) \sin^2 \omega_0 t + \frac{2e\omega_H}{\omega_0^2 - \omega_H^2} (p_x E_{0y} - p_y E_{0x}) \sin \omega_0 t + (2m_e k T_e) v_{ei} t]\}. \end{aligned}$$

The functions  $F_e^{0(1)}$  and  $F_e^{1(1)}$  in equations (5) and (6) are solutions to the inhomogeneous equation (3A) and (4A). Before finding them, consider the following characteristics of the homogeneous equation (2):

$$\frac{d\vec{p}}{dt} = -\vec{F}_0; \quad m_e \frac{d\vec{r}}{dt} = \vec{p}. \quad (7)$$

The solutions of these characteristic equations relate the values of momentum  $p_x, p_y, p_z$  and coordinates  $r_x, r_y, r_z$  of electrons at time  $t$  to the values  $P_{0x}, P_{0y}, P_{0z}$  and  $R_x, R_y, R_z$  at the initial time,  $t'$  i.e.

$$P_X = P_{0x} \cos \omega_H (t - t') - P_{0y} \sin \omega_H (t - t') + N_1 e (\omega_0 N_2 E_{0x} + N_3 E_{0y});$$

$$P_Y = P_{0x} \sin \omega_H (t - t') - P_{0y} \cos \omega_H (t - t') + N_1 e (\omega_0 N_2 E_{0y} - N_3 E_{0x});$$

$$\begin{aligned}
 P_z &= P_{0z} - e\omega_0^{-1}E_{0z} \{1 - \cos \omega_0(t-t')\}; \\
 r_x &= R_x - (m_e \omega_H)^{-1} P_{0y} \{1 - \cos \omega_H(t-t')\} + (m_e \omega_H)^{-1} P_{0x} \sin \omega_H(t-t') + m_e^{-1} e N_1 (N_4 E_{0y} + N_5 E_{0x}); \\
 r_y &= R_y - (m_e \omega_H)^{-1} P_{0x} \{1 - \cos \omega_H(t-t')\} + (m_e \omega_H)^{-1} P_{0y} \sin \omega_H(t-t') + m_e^{-1} e N_1 (N_5 E_{0y} - N_4 E_{0x}); \\
 r_z &= R_z + m_e^{-1} P_{0z}(t-t') - (m_e \omega_0)^{-1} e E_{0z} \{(t-t') - \omega_0^{-1} \sin \omega_0(t-t')\}; \\
 N_1 &= (\omega_0^2 - \omega_H^2)^{-1}; \\
 N_2 &= \cos \omega_0(t-t') - \cos \omega_H(t-t'); \\
 N_3 &= \omega_0 \sin \omega_H(t-t') - \omega_H \sin \omega_0(t-t'); \\
 N_4 &= \omega_0 \omega_H^{-1} \{1 - \cos \omega_H(t-t')\} - \omega_H \omega_0^{-1} \{1 - \cos \omega_0(t-t')\}; \\
 N_5 &= \sin \omega_0(t-t') - \omega_0 \omega_H^{-1} \sin \omega_H(t-t').
 \end{aligned}$$

When studying kinetic effects in inhomogeneous plasma, it is required to know the perturbed distribution function of such plasma. One way to find this function is the trajectory integral method, which was first used in problems of oscillations of homogeneous and inhomogeneous plasma in a magnetic field. Transitioning from  $\vec{p}$  to  $\vec{P}_0$  and as well as from  $\vec{r}$  to  $\vec{R}$  using the method mentioned above, we determine the desired solutions  $F_e^{0(1)}$  and  $F_e^{1(1)}$  in the form [17]:

$$F_e^{0(1)} = \int_0^1 v_{ei} \langle F_e^{0(0)}(t-t', \varepsilon t, \varepsilon \vec{R}, \vec{P}_0) \rangle dt'; \quad (8)$$

$$F_e^{1(1)} = - \int_0^1 S_1(t-t', \varepsilon t, \varepsilon \vec{R}, \vec{P}_0) dt'. \quad (9)$$

It should be noted that when integrating the integrand exponential functions in (8) and (9), the conditions were taken into account

$$\frac{eE_{0x}}{P_{0x}} < \Delta\omega; \quad \frac{eE_{0y}}{P_{0y}} < \Delta\omega; \quad \frac{eE_{0z}}{P_{0z}} < \Delta\omega,$$

under which these functions were expanded into a series. In this case, only terms were considered  $\sim E_0^2$ . Subsequently, by averaging the function  $F_e$  the following expressions for the Miller force components are obtained:

$$\begin{aligned}
 f_x &= \int_{-\infty}^{+\infty} T^{-1} P_{0x} \langle F_e \rangle d\vec{P}_0 = -N_1^2 n_e \frac{(e\omega_0)^2}{2m_e} \frac{\partial}{\partial \varepsilon R_x} (M_1^0 E_{0x}^2 + M_2^0 E_{0y}^2) - \frac{n_e}{2m_e} \left(\frac{e}{\omega_0}\right)^2 \frac{\partial E_{0z}^2}{\partial \varepsilon R_x} C_1; \\
 f_y &= \int_{-\infty}^{+\infty} T^{-1} P_{0y} \langle F_e \rangle d\vec{P}_0 = -N_1^2 n_e \frac{(e\omega_0)^2}{2m_e} \frac{\partial}{\partial \varepsilon R_y} (M_1^0 E_{0x}^2 + M_2^0 E_{0y}^2) - \frac{n_e}{2m_e} \left(\frac{e}{\omega_0}\right)^2 \frac{\partial E_{0z}^2}{\partial \varepsilon R_y} C_1; \\
 f_z &= \int_{-\infty}^{+\infty} T^{-1} P_{0z} \langle F_e \rangle d\vec{P}_0 = -N_1^2 n_e \frac{(e\omega_0)^2}{2m_e} \frac{\partial}{\partial \varepsilon R_z} (M_3^0 (E_{0x}^2 + E_{0y}^2)) - \frac{n_e}{2m_e} \left(\frac{e}{\omega_0}\right)^2 \frac{\partial E_{0z}^2}{\partial \varepsilon R_z} C_2,
 \end{aligned}$$

here

$$\begin{aligned}
 M_1^0 &= \frac{3}{2} \cos\left(2\pi \frac{\omega_H}{\omega_0}\right) - \frac{1}{3} \left\{ 2Q + \pi^{-2} \left(1 + v_{ei}^2 (4\omega_0^2 + v_{ei}^2)^{-1}\right) D \sin\left(2\pi \frac{\omega_H}{\omega_0}\right) \right\}; \\
 M_3^0 &= \frac{3}{2} \cos\left(2\pi \frac{\omega_H}{\omega_0}\right) - \frac{1}{3} \left\{ 2Q + \frac{5}{2} \pi^{-2} \left(1 + v_{ei}^2 (4\omega_0^2 + v_{ei}^2)^{-1}\right) D \sin\left(2\pi \frac{\omega_H}{\omega_0}\right) \right\}; \\
 M_2^0 &= \frac{1}{2} + \cos\left(2\pi \frac{\omega_H}{\omega_0}\right) - \frac{1}{3} \left\{ 2Q + \frac{1}{2} \pi^{-2} \left(1 + v_{ei}^2 (4\omega_0^2 + v_{ei}^2)^{-1}\right) D \sin\left(2\pi \frac{\omega_H}{\omega_0}\right) \right\};
 \end{aligned}$$

$$\begin{aligned}
 C_1 &= \frac{1}{2} \left( \cos \left( 2\pi \frac{\omega_H}{\omega_0} \right) + 2 \right) - \frac{1}{3} \left\{ 2Q + \frac{1}{2} \pi^{-2} v_{ei}^2 (4\omega_0^2 + v_{ei}^2)^{-1} D \sin \left( 2\pi \frac{\omega_H}{\omega_0} \right) \right\}; \\
 C_2 &= \frac{1}{2} \left( \cos \left( 2\pi \frac{\omega_H}{\omega_0} \right) + 2 \right) - \frac{1}{3} \left\{ 2Q + \frac{3}{2} \pi^{-2} v_{ei}^2 (4\omega_0^2 + v_{ei}^2)^{-1} D \sin \left( 2\pi \frac{\omega_H}{\omega_0} \right) \right\}; \\
 Q &= \frac{\omega_0}{2\pi} \left[ (\omega_0 - \omega_H)^2 + v_{ei}^2 \right]^{-1} \left\{ v_{ei} - \left[ (\omega_0 - \omega_H) \sin \left( 2\pi \frac{\omega_H}{\omega_0} \right) + v_{ei} \cos \left( 2\pi \frac{\omega_H}{\omega_0} \right) \right] (1 - D) \right\}; \\
 D &= 1 - \exp(-2\pi v_{ei} / \omega_0).
 \end{aligned}$$

In the absence of a constant magnetic field ( $\vec{B}_0 \rightarrow 0$ ) or under the condition  $(\omega_H / \omega_0) < \varepsilon$  the following expression for the force is obtained  $\vec{f}_M$ :

$$\vec{f}_M = -\frac{3}{2} \frac{n_e}{2m_e} \left( \frac{e}{\omega_0} \right)^2 \frac{\partial \vec{E}_0^2}{\partial \varepsilon \vec{R}} \left\{ 1 - \frac{4}{9} \left( \frac{v_{ei}}{\omega_0} \right)^2 \right\}. \quad (10)$$

If we neglect collisions ( $v_{ei} \rightarrow 0$ ), then from (10) we have  $\vec{f}_M = -\frac{3}{2} n_e \nabla_{\varepsilon \vec{R}} U_e$ , where  $\nabla_{\varepsilon \vec{R}}$  is nabla. Thus, an expression for the quasi-potential Miller force has been derived, taking into account for electron-ion collisions. The obtained expression, in the limiting cases  $\vec{B}_0 \rightarrow 0$  and  $v_{ei} \rightarrow 0$  coincides with known expression for the high-frequency pressure force up to a coefficient of order unity [18].

### Conclusion

An expression for the quasi-potential force in the approximation of fixed ions and a strong external field has been obtained. This expression allows us to estimate the influence of collisions of plasma particles on the Miller force. The force components have been determined for the case when a high-frequency electric field and a stationary magnetic field acting simultaneously on weakly inhomogeneous plasma. Thus, this article introduces a new methodology for determining the expression of the averaged high-frequency pressure force. The approach is based on solving the kinetic equation and the method of successive approximations while observing limiting conditions. The motion of charged particles in an electric field is considered based on the concepts of classical physics, and these concepts retain their validity not only when analyzing the motion of charged particles under the influence of macroscopic external fields but also form the foundation necessary for understanding the processes of particle interaction in plasma — processes involving the microscopic fields of individual particles.

*Practical significance of research results.* The results of this article can be applied in controlling the spatial distribution of plasma parameters in external fields and in the mathematical modeling of inhomogeneous plasma processes. Furthermore, these results can be applied in the theory of electron motion in high-frequency and constant fields, fluctuations, nonequilibrium processes, the stability of inhomogeneous plasma, and other collective nonlinear phenomena.

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## Тұрақты магнит өрісі мен жоғары жиілікті электр өрісінің плазмаға әсері

Мақалада әлсіз біртекті плазмаға біртекті жоғары жиілікті электр өрісі мен тұрақты магнит өрісі әсер еткен жағдай үшін орташаланған жоғары жиілікті қысым күшін (Миллер күшін) анықтаудың тың тәсілі ұсынылған. Бұл тәсіл электрондардың үлестірім функциясына арналған кинетикалық теңдеуді тізбектестік жуықтау арқылы шешуге негізделген. Аталған жуықтау бойынша баяу қозғалыстар мен қарқынды тербелістер бөлек қарастырылады. Сонымен бірге, есептеулер кезінде теориялық және математикалық физиканың жалпыға белгілі тәсілдері, атап айтқанда сыртқы айнымалы өрістің периоды бойынша орташалау және траектория бойынша интегралдау әдістері де пайдаланылды. Электрондардың қозғалмайтын иондармен соқтығысулары ескерілді. Жоғары жиілікті өрістің амплитудасы уақыт және координата бойынша мардымсыз ғана өзгеретін функция. Шығарып алынған өрнек плазма бөлшектерінің өзара соқтығысулары Миллер күшіне әсерін бағалауға мүмкіндік жасайды және белгілі бір шектеуіш шарттарда бұл өрнек электрондардың жоғары жиілікті өрістердегі қозғалысын сипаттайтын теңдеу арқылы анықталған белгілі өрнекпен сәйкес келеді. Есептеулер кезінде сыртқы электромагниттік өрістің магниттік құраушысы ескерілген жоқ, бұл жағдай жоғары жиілікті электр өрісінің бойлық (қума) екендігін айғақтайды. Аталып өтілген барлық нәтижелер теориялық сипатта болғандықтан оларды жоғары жиілікті электромагниттік өрістердің әсеріндегі біртекті плазманың кинетикалық теориясын жасау барысында қолдануға болады.

*Кілт сөздер:* әлсіз біртекті плазма, плазмалық электрондар, кинетикалық теңдеу, орташаланған күш, электр өрісі, бөлшектердің соқтығысуы, қозғалмайтын иондар, жоғары жиілік.

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## Воздействие постоянного магнитного поля и высокочастотного электрического поля на плазму

В данной работе предложена новая методика определения выражения усредненной силы высокочастотного давления (силы Миллера). Этот метод основан на решении кинетического уравнения для функции распределения электронов и методе последовательных приближений (разделение медленных движений и быстрых осцилляций). Рассматривается случай, когда на слабо неоднородную плазму одновременно действуют электрическое поле высокой частоты и стационарное магнитное поле. В исследовании также применены общеизвестные методы теоретической и математической физики, такие как усреднение по периоду колебания электрического поля и интегрирование по траекториям. Учтены столкновения электронов с неподвижными ионами. Амплитуда электрического поля является медленно меняющейся функцией по времени и по координатам. Полученное выражение позволяет оценить влияние столкновений плазменных частиц на силу Миллера и подтвердить, что при определенных условиях оно совпадает, с точностью до постоянной, с известными выражениями для силы высокочастотного давления, полученными на основе уравнения движения электронов плазмы в высокочастотных полях. Во всех вычислениях пренебрегается вкладом магнитной составляющей электромагнитного поля, что вполне справедливо для продольного электрического поля. Результаты, полученные в данном исследовании, представляют прежде всего теоретический интерес и раскрывают механизмы взаимодействия слабо неоднородной плазмы с высокочастотным электрическим полем. Они также могут быть использованы при построении кинетической теории неоднородной плазмы, находящейся в высокочастотных электромагнитных полях.

*Ключевые слова:* слабо неоднородная плазма, плазменные электроны, кинетическое уравнение, усредненная сила, электрическое поле, столкновения частиц, неподвижные ионы, высокая частота.

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