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# Introduction of a New B3Y-Fetal Potential in the Semimicroscopic Analysis of the $^{15}$N + $^{27}$Al Nuclear System

The experimental data analysis of the low-energy elastic scattering process in the $^{15}$N+$^{27}$Al nuclear system used a new microscopic approach. In the microscopic analysis, new B3Y-Fetal potentials, calculated using the variational method with lower-order constraints (LOCV) in two-body matrices, were applied. Based on the double folding model (DFM), the CDM3Y2-Paris, CDB3Y2-Fetal, and CDB3Y3-Fetal microfolding potentials were constructed by adjusting density-dependent parameters $C$, α, β, and γ. These density-dependent parameters were introduced based on the effective nucleon-nucleon (NN) interaction and the form factor of the nucleon density distribution of the colliding $^{15}$N and $^{27}$Al nuclei. The uniqueness of the analysis lies in the fact that the introduced density-dependent parameters were calculated using the optimal value of the $K$ — compressibility factor, which characterizes the saturation properties of the nuclear medium. The optimal parameter sets for the optical and folding potentials were determined from the results of the optical model (OM) and DFM analyses. The theoretical cross-sections of elastic scattering obtained from phenomenological and microscopic analyses were found to describe the experimental data well. In the semi-microscopic analysis, the effectiveness of the B3Y-Fetal folding potential was determined. Based on the analysis of the experimental data of elastic scattering, it was concluded that the saturation properties of nuclear matter can be determined more accurately. The low-energy elastic scattering reaction in the $^{15}$N+$^{27}$Al nuclear system is important for studying the properties of materials and nuclear fuels. This study of the nuclear process contributes to the development of future nuclear materials and energy technologies.

*Keywords:* microscopic analysis, elastic scattering, double folding model, B3Y-Fetal potential, nucleon density distribution.

## *Introduction*

Studying the processes occurring during nuclear collisions helps to obtain valuable information about nuclear forces and their behavior, understand nuclear reaction mechanisms, and the structure of complex nuclei. Research on $^{15}$N+$^{27}$Al nuclear systems has been actively conducted in recent years from both fundamental and applied science perspectives. Such studies make it possible to determine the energy levels and states of nuclei, which is important for the theoretical modeling of nuclear interactions.

Studying reactions between light elements such as $^{15}$N and $^{27}$Al helps to understand nucleosynthesis processes and model the evolution of stars, which is important for nuclear astrophysics. In nuclear energy, processes involving light nuclei like nitrogen and aluminum under thermonuclear synthesis conditions are considered as a potential future energy source. Understanding the effect of thermonuclear reaction products and fast neutrons on the high-energy characteristics of materials containing the $^{27}$Al nucleus is one of the key aspects. Analyzing the $^{15}$N+$^{27}$Al reaction aids in the development of radiation detectors and monitoring devices for nuclear reactors. Thus, the $^{15}$N+$^{27}$Al system has a wide range of practical applications in nuclear energy, from fundamental research to technology and safety.

In addition to obtaining important information about nuclear structure and reaction mechanisms through the analysis of the elastic scattering process, it is possible to enhance the saturation properties of the nuclear medium. The saturation point of the binding energy density dependence, $K$ — nuclear incompressibility, is directly related to the nuclear binding energy [1]. The approach to constructing the equation of state of a nucleus based on the overlap of nucleon densities of interacting nuclei is performed through a microscopic method. Constructing an effective NN-interaction potential dependent on nucleon density enables the calculation of the saturation properties of nuclear matter [2]. The novelty of this work lies in investigating the efficiency of using the effective NN-interaction B3Y-Fetal potential with density-dependent parameters $C$, α, β, γ. The effective NN-interaction B3Y-Fetal potential of Fiase, incorporating these density-dependent parameters, was recently obtained based on calculating nuclear matrix elements of two-body interactions using the LOCV method [3–6].

From this, an analysis was carried out on the basis of density — dependent parameters calculated at the value $\rho_0 = 0.17$ fm$^{-3}$ of the actual density determined at the saturation point. The density-dependent α-parameters are determined in the effective NN-interaction formula in the DFM formula [7].

$$t_i\left(r_{NN}\right) = C_i\left[1+\alpha_i\rho^{2/3}_{P(T)}\left(S_{P(T)}\right)\right]V^i_{KK}\left(r_{NN}\right), \tag{1}$$

where $V^i_{KK}(r_{NN})$ is the potential of singlet and triplet states.

In the articles Dao Khoa, 2 types of *F*(*p*) — density-dependent form factor are proposed [8, 9]. The article identified density-dependent parameters *C,* α, β, γ — at optimal values of the *K*-incompressibility factor and analyzed in a wide energy range [9–12].

The folding model allows modification of the density-dependent form factor of nucleon distributions taking into account the ideas of microscopic theory. Therefore, the properties of nuclear matter can be described more clearly using effective NN-interaction. No such analysis has been done for system $^{15}$N+$^{27}$Al. In our upcoming work [13, 14], analyses were made using the new B3Y-Fetal potential that resulted in global dependencies.

*Calculation of the K-incompressibility factor depending on the saturation property of the nucleus*

To increase the saturation property of nuclear matter, the equilibrium condition of the density-dependent specific bond energy is sufficient. We apply density-dependent parameters calculated for the equilibrium condition to the effective NN-interaction M3Y potential. In particular, it is necessary to construct the equations of state at the value $\rho_0 = 0.17$ fm$^{-3}$ of the saturation density of nuclear matter. Specific binding energy of density-dependent infinite nuclear matter [4, 8]:

$$\frac{\varepsilon_0}{A}(\rho) = \frac{3\hbar^2k_F^2}{10m} + F(\rho)\frac{\rho}{2}\left(J_D + \int\left[\hat{j}_1\left(k_F r\right)\right]^2 \upsilon^{EX}\left(r\right)d^3r\right), \tag{2}$$

where *m* is the mass of the nucleon; $J_D$ is the integral obtained by volume from the direct part of the interaction $\hat{j}_1\left(x\right) = \frac{3\mathrm{j}1\left(x\right)}{X}$ — Bessel spherical function ($n = 1$), ρ is the density of the nucleons, $k_F$ is the Fermi momentum.

From the equilibrium saturation condition of the binding energy of the nucleus:

$$\frac{d\varepsilon}{d\rho} = \frac{\hbar^2k_F^2}{5mp} + \frac{J_D C}{2}\left(1-\frac{5}{3}\beta\left(\varepsilon\right)\rho^{2/3}\right) = 0. \tag{3}$$

Based on the density-dependent parameters from Equations (4) and (3), the ε — bond energy formula is written as follows:

$$\varepsilon = \frac{3\hbar^2k_F^2}{10m} + \frac{\rho J_D}{2}C\left(1-\beta\left(\varepsilon\right)\rho^{2/3}\right). \tag{4}$$

Density-dependent *c*, β — parameters and $\rho_0$ — actual (saturation) density [4]:

$$C = -\frac{2\hbar^2k_F^2}{5mJ_D\rho\left(1-\frac{5}{3}\beta\left(\varepsilon\right)\rho^{\frac{2}{3}}\right)} \tag{5}$$

$$\beta\left(\varepsilon\right) = \frac{\left(3-3p\right)}{\left(9-5p\right)}\cdot\frac{1}{\rho^{5/3}}, \tag{6}$$

$$\rho_0 = -\frac{10m\varepsilon}{\hbar^2(1.5\pi^2\rho)^{2/3}}. \tag{7}$$

The nuclear incompressibility factor is calculated by the following equation [4]:

$$K_\infty = \left[-\frac{3\hbar^2k_F^3}{5m} + 5J_D C\beta\left(\varepsilon\right)\rho^{5/3}\right]_{\rho=\rho_0}. \tag{8}$$

The saturation density point ($\rho_0 = 0.17$ fm$^{-3}$) defined according to the specific binding energy of the nucleus using the B3Y-Fetal interactionand shown in Figure 1 [4].

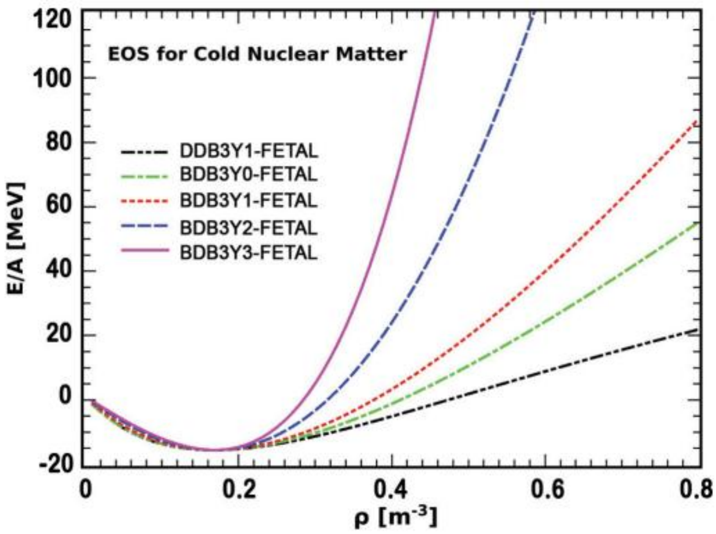


Figure 1. Saturation point of the core binding energy density dependence [4]

*NN-interaction on effective forces M3Y-Paris and B3Y-Fetal potentials*

The interaction potential for effective NN forces consists of the sum of the $U^D(\vec{R})$ — direct and $U^{EX}(\vec{R})$ — exchange potentials.

$$U(\vec{R}) = U^D(\vec{R}) + U^{EX}(\vec{R}). \tag{9}$$

Double integrated direct potential on the distribution of nucleons of beam and target nuclei [15, 16]:

$$U^D(\vec{R}) = \iint \rho^{(1)}(r_1)\upsilon_D(s)\rho^{(2)}(r_2)dr_1 dr_2, \tag{10}$$

where $\upsilon_D(\vec{s})$ — direct component of effective interaction; $\rho^{(i)}(r_i)$ — nucleon density of colliding nuclei; $s$ — effective NN-interaction distance, $s = r_2 - r_1 + R$.

Double integrated Exchange potential on the distribution of nucleons of beam and target nuclei [16, 17]:

$$U^{EX}(\vec{R}) = \iint \rho^{(1)}(\vec{r}_1, \vec{r}_1 + \vec{s})\upsilon_{EX}(\vec{s})\rho^{(2)}(\vec{r}_2, \vec{r}_2 - \vec{s})\exp\left[i\vec{k}(\vec{R})\vec{s}/\eta\right]d\vec{r}_1 d\vec{r}_2, \tag{11}$$

where $\upsilon_{EX}(\vec{s})$ — effective NN-interaction exchange component, $\rho^{(i)}(\vec{r}, \vec{r}')$ — density matrix of colliding nuclei.

Matrix in singlet, triplet states of nucleon interaction (Hartree Fock) calculation for coefficients direct and transition components [18]:

$$\upsilon_{D(EX)} = 1/16\left(3\upsilon_{TE}^c + 3\upsilon_{SE}^c \pm 9\upsilon_{T0}^c \pm \upsilon_{S0}^c\right), \tag{12}$$

where the triplet and singlet components of the central forces are even $\left(\upsilon_{TE}^c, \upsilon_{SE}^c\right)$ and odd $\left(\upsilon_{T0}^c, \upsilon_{S0}^c\right)$ [19].

Direct and exchange components based on the G-matrix element of the M3Y-Paris potential [20].

$$\upsilon_D(s) = 11061.6\frac{\exp(-4s)}{4s} - 2537.5\frac{\exp(-2.5s)}{2.5s}, \tag{13}$$

$$\upsilon_D(s) = -1524.0\frac{\exp(-4s)}{4s} - 518.8\frac{\exp(-2.5s)}{2.5s} - 7.8474\frac{\exp(-0.7072s)}{0.7072s}. \tag{14}$$

The direct and transition components based on the *G*-matrix element of the B3Y-Fetal potential is given in the radial form of the isoscalar part [6, 14]:

$$\upsilon_D(s) = 10472.13\frac{\exp(-4s)}{4s} - 2203.11\frac{\exp(-2.5s)}{2.5s}, \tag{15}$$

$$\upsilon_{EX}(s) = 499.63\frac{\exp(-4s)}{4s} - 1347.77\frac{\exp(-2.5s)}{2.5s} - 7.8474\frac{\exp(-0.7072s)}{0.7072s}. \tag{16}$$

*The theory of semi-microscopic analysis*

When analyzing experimental data of elastic scattering within the framework of an optical model (OM), the Woods-Saxon form of potential was used.

$$U(r)=V_o\left[1+\exp\left(\frac{r-R_V}{a_V}\right)\right]-iW_o\left[1+\exp\left(\frac{r-R_W}{a_W}\right)\right]+V_C(r), \tag{17}$$

where $V_o$, $W_V$, $a_V$, $a_W$, $R_V$, $R_W$ are real, imaginery potentials, diffusion, radius, $V_C(r)$ is the Coulomb potential.

In a semi-microscopic analysis, we replace the real part of the op with the $V_F(r)$ — folding potential:

$$U(r)=N_r\left[V_D(r)+V_{EX}(r)\right]-iW_o\left[1+\exp\left(\frac{r-R_W}{a_W}\right)\right]+V_C(r), \tag{18}$$

where $N_r$ is a re-rationing factor of the folding potential.

The folding potential of an effective NN-interaction [17]:

$$V_F=\iint\rho_1(r_1)\rho_2(r_2)\vartheta_{NN}(s)d^3r_1d^3r_2, \tag{19}$$

where $\vartheta_{NN}$ is the effective NN-interaction potential; $\rho_1(r_1)$ and $\rho_2(r_2)$ is the distribution of the matter density of the beam and target nuclei, respectively.

$V_C(r)$ — the Coulomb potential is defined as follows:

$$V_C(R)=\begin{cases}\dfrac{Z_1Z_2e^2}{2R_C}\left(3-\dfrac{R^2}{R_C}\right) & \text{For} \quad R\le R_C\\ \dfrac{Z_1Z_2e^2}{R} & \text{For} \quad R\ge R_C\end{cases} \tag{20}$$

For the $^{15}$N-core, the harmonic-oscillator model was selected as the distribution of the density of matter [21]:

$$\rho(r)=\rho_0\left(1+\alpha(r/a)^2\right)\exp\left(-(r/a)^2\right), \tag{21}$$

where $\alpha$ = 1.756 fm, $a$ = 1.29 fm [22].

For the $^{27}$Al core, the two-parameter Fermi modelwas selected as the distribution of the density of matter [21]:

$$\rho(r)=\rho_0\Big/\left(1+\exp\left(\frac{r-c}{z}\right)\right), \tag{22}$$

where $c$ = 3.07 fm, $z$ = 0.519 fm [22].

Dependence function of direct and exchange potentials [8].

$$\upsilon_{D(EX)}(\rho,r)=g(E)f(\rho)\upsilon'_{D(EX)}(r), \tag{23}$$

where $g(E)$ — energy dependent type; $\rho$ — density of the overlapping medium of nucleons of nuclei; $r$ is the distance between the interacting nucleons.

In the process of elastic scattering, there is a re-distribution of energy between the colliding nuclei. Energy dependent type [13].

$$g(E)=1-0.003(E/A). \tag{24}$$

A type proposed by Dao Khoa of introducing density-dependent $C$, $\alpha$, $\beta$ — parameters to the effective NN-interaction [6]:

$$1)\ f(\rho)=C\left(1+\alpha e^{-\beta\rho}\right), \quad \text{the DDM3Y-type,} \tag{25}$$

$$2)\ f(\rho)=C\left(1-\alpha\rho^{\beta}\right), \quad \text{the BDM3Y-type.} \tag{26}$$

Based on this $f(p)$ — density-dependent form factor, you can enter density-dependent parameters for the correct values of $K$-incompressibility [8, 13]. $\gamma$ — parameter dependent formula [6, 13]:

$$f(\rho)=C\left(1+\alpha\exp(-\beta\rho)-\gamma\rho\right). \tag{27}$$

Table 1

**Density-dependent modified microfolding potentials [6, 22]**

| Density dependence | *C* | α | β ($fm^3$) | γ ($fm^3$) | *K* (MeV) |
|---|---|---|---|---|---|
| CDM3Y2-Paris<br>CDB3Y2-Fetal | 0.3346 | 3.0357 | 3.0685 | 1.0 | 204 |
| CDB3Y3-Fetal | 0.2985 | 3.4528 | 2.6388 | 1.5 | 217 |

*Discussion of results*

At energies $E_{Lab}$ = 33 MeV, $E_{Lab}$ = 48 MeV, $E_{Lab}$ = 62 MeV, $E_{Lab}$ = 70 MeV [23], for the nuclear system $^{15}N+^{27}Al$, phenomenolic and semi-microscopic analyses of experimental data of elastic scattering were performed.

The analysis used density-dependent CDM3Y2-Paris, CDB3Y2-Fetal and CDB3Y3-Fetal folding potentials. All OM and DFM calculations were carried out on the basis of using the Fresco Code [24]. The following figure shows the result of the OM and DFM analyzes and shown in Figures 2–5.

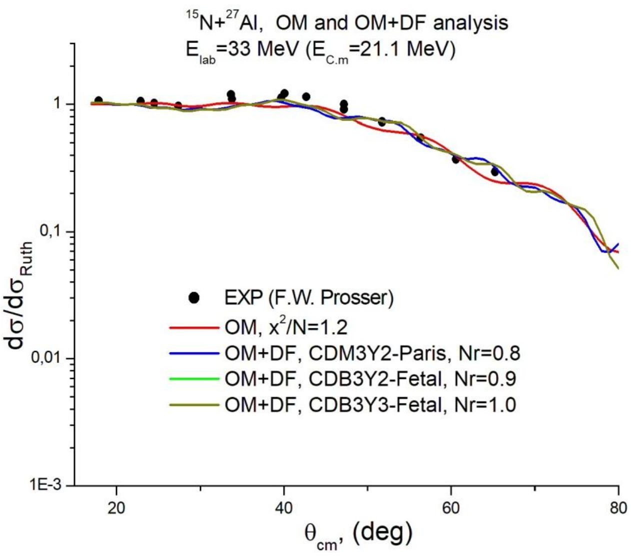


Figure 2. Results of the analysis of OM and DFM

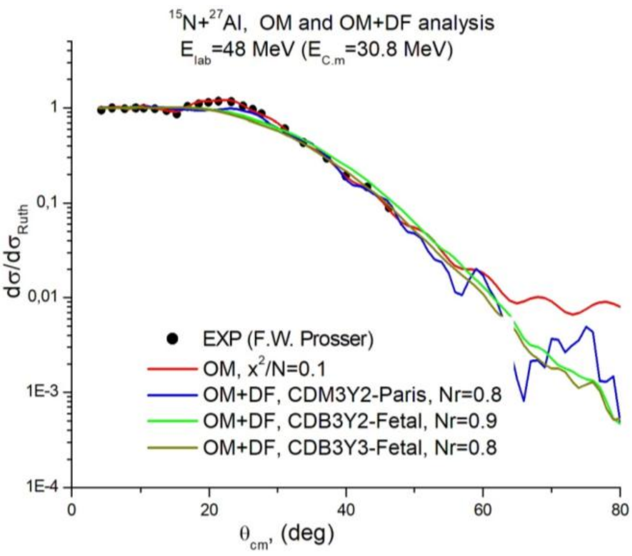


Figure 3. Results of the analysis of ohms and DFM

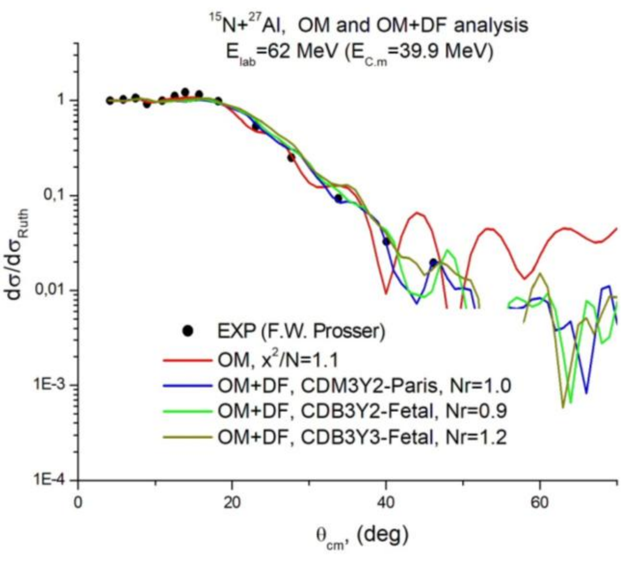


Figure 4. Results of the analysis of ohms and DFM

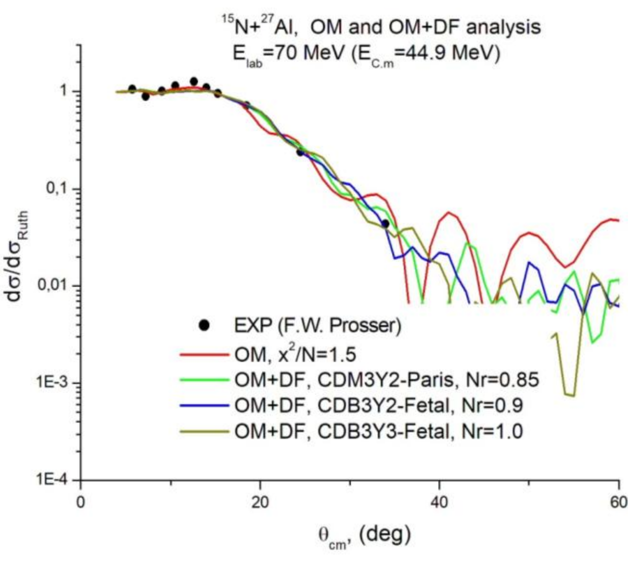


Figure 5. Results of the analysis of OM and DFM

Microfolding potentials CDM3Y2-Paris, CDB3Y2-Fetal, CDB3Y3-Fetal were used in the OM+DF analysis. The $N_r$ — re-rationing factor was determined at intervals of 0.8–1.2.

The values of the optimal parameters for theoretical analysis are shown in Table 2. There is an energy dependence of the characteristics of the parameters.

Table 2

**Parameters of OM and DFM analysis for $^{15}N+^{27}Al$ system**

| $E$ (MeV) | Model | Type of real potential | | | $N_R$ | Imaginary potential parameter (WS) | | | $\sigma_R$ (mb) | $\chi^2/N$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $V_0$ (MeV) | $Rv$ (fm) | $av$ (fm) | | $W_0$ (MeV) | $r_W$ (fm) | $a_W$ (fm) | | |
| 33 | OM | 280.0 | 0.97 | 0.5 | – | 4.6 | 1.2 | 1.15 | 1093 | 1.2 |
| | OM+DF | CDM3Y2-Paris | | | 0.8 | 4.6 | 1.2 | 1.15 | 1085 | – |
| | OM+DF | CDB3Y2-Fetal | | | 0.9 | 4.6 | 1.2 | 1.17 | 1055 | – |
| | OM+DF | CDB3Y3-Fetal | | | 1.0 | 4.6 | 1.2 | 1.17 | 1055 | – |
| 48 | OM | 240.8 | 1.13 | 0.5 | – | 24.9 | 1.37 | 0.2 | 1552 | 0.1 |
| | OM+DF | CDM3Y2-Paris | | | 0.8 | 24.9 | 1.37 | 0.2 | 1602 | – |
| | OM+DF | CDB3Y2-Fetal | | | 0.9 | 24.9 | 1.37 | 0.2 | 1570 | – |
| | OM+DF | CDB3Y3-Fetal | | | 0.8 | 24.9 | 1.37 | 0.2 | 1554 | – |
| 62 | OM | 279.9 | 1.04 | 0.47 | – | 4.6 | 1.24 | 1.17 | 1834 | 0.1 |
| | OM+DF | CDM3Y2-Paris | | | 1.0 | 4.6 | 1.24 | 1.17 | 1848 | – |
| | OM+DF | CDB3Y2-Fetal | | | 0.9 | 4.6 | 1.24 | 1.17 | 1782 | – |
| | OM+DF | CDB3Y3-Fetal | | | 1.2 | 4.6 | 1.24 | 1.17 | 1820 | – |
| 70 | OM | 240.8 | 1.13 | 0.5 | – | 24.9 | 1.37 | 0.2 | 1904 | 0.5 |
| | OM+DF | CDM3Y2-Paris | | | 0.85 | 24.9 | 1.37 | 0.2 | 1894 | – |
| | OM+DF | CDB3Y2-Fetal | | | 0.9 | 24.9 | 1.37 | 0.2 | 1885 | – |
| | OM+DF | CDB3Y3-Fetal | | | 1.0 | 24.9 | 1.37 | 0.2 | 1910 | – |

## *Conclusion*

• A semi-microscopic analysis was carried out for the $^{15}N+^{27}Al$ system using the new B3Y-Fetal potential calculated on the basis of LOCV.

• Density-dependent parameters $C$, α, β, γ were introduced into the new B3Y-Fetal potential, and the CDM3Y2-Paris, CDB3Y2-Fetal, and CDB3Y3-Fetal folding potentials were created. Theoretical elastic scattering cross sections have been determined for the $^{15}N+^{27}Al$ system.

• The efficiency of the microscopic CDB3Y-Fetal potential was determined for the $^{15}N+^{27}Al$ system. The accuracy of the analysis of OM was determined in the range of $\chi^2/N = 0.1–1.8$. The reordering coefficient of semi-microscopic analysis was determined at the interval $N_r = 0.8–1.2$.

• Density-dependent study of the B3Y-Fetal potential, created on the basis of LOCV made it possible to clarify the saturation property of nuclear matter, to fully take into account nuclear forces.

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## References

1. Ochala, I., Fiase, J.O., & Anthony, E. (2017). Computation of nuclear binding energy and incompressibility with a new M3Y-type effective interaction. *Int Res J Pure Applied Physics*, *5*, 5–13.
2. Khoa, D.T., Satchler, G.R., & Von Oertzen, W. (1997). Nuclear incompressibility and density dependent NN interactions in the folding model for nucleus-nucleus potentials. *Physical Review C*, *56*(2), 954. https://doi.org/10.1103/PhysRevC.56.954.
3. Fiase, J.O., Devan, K.R.S., & Hosaka, A. (2002). Mass dependence of M3Y-type interactions and the effects of tensor correlations. *Physical Review C*, *66*(1), 014004. https://doi.org/10.1103/PhysRevC.66. 014004.
4. Ochala, I., & Fiase, J.O. (2018). Symmetric nuclear matter calculations: A variational approach. *Physical Review C*, *98*(6), 064001. https://doi.org/10.1103/PhysRevC.98.064001.
5. Ochala, I., Fiase, J.O., Gbaorun, F., & Bamikole, J.A. (2021). A Study of Asymmetric Nuclear Matter with the B3Y-Fetal Effective Interaction. *International Research Journal of Pure and Applied Physics*, *8*(2), 10–35. https://doi.org/10.37745/irjpap.13.

6. Ochala, I., & Fiase, J.O. (2021). B3Y-FETAL effective interaction in the folding analysis of elastic scattering of $^{16}O+^{16}O$. *Nuclear Science and Techniques*, *32*(8), 81. https://doi.org/10.1007/s41365-021-00920-z.
7. Majka, Z., Gils, H.J., & Rebel, H. (1978). 104 MeV alpha particle and 156 MeV6Li scattering and the validity of refined folding model approaches for light complex projectile scattering. *Zeitschrift für Physik A Atoms and Nuclei*, *288*(2), 139–152.
8. Khoa, D.T., Von Oertzen, W., & Bohlen, H.G. (1994). Double-folding model for heavy-ion optical potential: Revised and applied to study C12 and O16 elastic scattering. *Physical Review C*, *49*(3), 1652. https://doi.org/10.1103/PhysRevC.49.1652.
9. Khoa, D.T., & Von Oertzen, W. (1995). Refractive alpha-nucleus scattering: a probe for the incompressibility of cold nuclear matter. *Physics Letters B*, *342*(1-4), 6–12. https://doi.org/10.1016/0370-2693(94)01393-Q.
10. Amangeldi, N., Burtebayev, N., Soldatkhan, D., Nassurlla, M., Mauyey, B., Yergaliuly, G., ... & Hamada, S. (2024). Recent Measurement and Theoretical Analysis for the Elastic Scattering of the $^{15}N+^{11}B$ System. *Brazilian Journal of Physics*, *54*(5), 169. https://doi.org/10.1007/s13538-024-01547-2
11. Amangeldi, N., Burtebayev, N., Artemov, S. V., Nassurlla, M., Mauyey, B., Yergaliuly, G., ... & Hamada, S. (2024). Efficiency of the new B3Y-fetal potential in the analysis of the elastic and inelastic angular distributions for the $^{10}B+^{12}C$ system. *Pramana*, *98*(3), 106. https://doi.org/10.1007/s12043-024-02760-z.
12. Soldatkhan, D., Amangeldi, N., Baltabekov, A.S., & Yergaliuly, G. (2022). Investigation of the energy dependence of the interaction potentials of the $^{16}O+^{12}C$ nuclear system with a semi-microscopic method. https://doi.org/10.31489/2022No3/39-44.
13. Soldatkhan, D., Yergaliuly, G., Amangeldi, N., Mauyey, B., Odsuren, M., Ibraheem, A.A., & Hamada, S. (2022). New Measurements and Theoretical Analysis for the $^{16}O+^{12}C$ Nuclear System. *Brazilian Journal of Physics*, *52*(5), 152. https://doi.org/10.1007/s13538-022-01153-0.
14. Soldatkhan, D., Amangeldi, N., Makhanov, K.M., Smagulov, Zh.K. (2023). Application of the new B3Y-Fetal potential in the semi-microscopic analysis of the scattering of accelerated $^{6}Li$ — lithium and $^{16}O$ — oxygen nuclei from the $^{12}C$ — carbon nucleus. *Eurasian Physical Technical Journal*, 25, 4(46), 22–30.
15. Khoa, D.T., & Satchler, G.R. (2000). Generalized folding model for elastic and inelastic nucleus–nucleus scattering using realistic density dependent nucleon–nucleon interaction. *Nuclear Physics A*, *668*(1–4), 3–41.
16. Kobos, A.M., Brown, B.A., Lindsay, R., & Satchler, G.R. (1984). Folding-model analysis of elastic and inelastic α-particle scattering using a density-dependent force. *Nuclear Physics A*, *425*(2), 205–232. https://doi.org/10.1016/0375-9474(84)90073-3.
17. Satchler, G.R. (2005). Folding models for elastic and inelastic scattering. In *Heavy-Ion Collisions: Proceedings of the International Summer School Held in Rábida (Huelva), Spain, June 7–18, 1982* (pp. 25–43). Berlin, Heidelberg: Springer Berlin Heidelberg.
18. Bertsch, G., Borysowicz, J., McManus, H., & Love, W.G. (1977). Interactions for inelastic scattering derived from realistic potentials. *Nuclear Physics A*, *284*(3), 399–419. https://doi.org/10.1016/0375-9474(77)90392-X.
19. Anantaraman, N., Toki, H., & Bertsch, G.F. (1983). An effective interaction for inelastic scattering derived from the Paris potential. *Nuclear Physics A*, *398*(2), 269–278.
20. Khoa, D.T., Phuc, N.H., Loan, D.T., & Loc, B.M. (2016). Nuclear mean field and double-folding model of the nucleus-nucleus optical potential. *Physical Review C*, *94*(3), 034612.
21. De Vries, H., De Jager, C.W., & De Vries, C. (1987). Nuclear charge-density-distribution parameters from elastic electron scattering. *Atomic data and nuclear data tables*, *36*(3), 495–536. https://doi.org/10.1016/0092-640X(87)90013-1.
22. Ochala, I., Gbaorun, F., Bamikole, J.A., & Fiase, J.O. (2019). A Microscopic Study of Nuclear Symmetry Energy with an Effective Interaction Derived from Variational Calculations. *Int Res J Pure Applied Physics*, *6*, 22–33.
23. Prosser Jr, F.W., Racca, R.A., Daneshvar, K., Geesaman, D.F., Henning, W., Kovar, D.G., ... & Tabor, S.L. (1980). Complete fusion of N15+Al27. *Physical Review C*, *21*(5), 1819.
24. Thompson, I.J. (1988). Getting started with FRESCO. *Comput. Phys. Rep*, *7*, 167–212.


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# $^{15}N + ^{27}Al$ ядролық жүйені жартылай микроскопиялық талдау кезінде жаңа B3Y-Fetal потенциалын енгізу

$^{15}N+^{27}Al$ ядролық жүйенің төмен энергиядағы серпімді шашырау процесінің эксперименттік деректерін талдауда жаңа микроскопиялық тәсіл қолданылды. Микроскопиялық талдауда екі дене матрицасында төменгі ретті шектеулері бар вариациялық тәсілде (LOCV) есептелген жаңа B3Y-Fetal потенциалдары пайдаланылды. Екілік фолдинг модель (DFM) негізінде тығыздыққа тәуелді *C*, α, β, γ — параметрлерді реттеу арқылы CDM3Y2-Paris, CDB3Y2-Fetal және CDB3Y3-Fetal микрофолдинг потенциалдары құрылды. Тығыздыққа тәуелді параметрлер тиімді нуклон-нуклондық (NN) өзара әрекеттесуі негізінде соқтығысқан $^{15}N$ және $^{27}Al$ ядролардың нуклондар тығыздығының таралуының форм-факторы бойынша енгізілді. Талдаудың ерекшелгі енгізілген тығыздыққа тәуелді параметрлер ядролық ортаның қанықтылық қасиетін сипаттайтын *K* — сығылмаушылық факторының оңтайлы мәнінде есептелген. Оптикалық модель (OM) және DFM талдаулар нәтижесінде оптикалық және фолдинг потенциалдардың оңтайлы параметрлер жиынтығы табылды. Феноменологиялық және микроскопиялық талдау нәтижелерінде серпімді шашыраудың теориялық қималары эксперименттік деректерді жақсы сипаттай алды. Жартылай микроскопиялық талдауда B3Y-Fetal фолдинг потенциалдың тиімділігі анықталды. Серпімді шашыраудың эксперименттік деректерін талдау негізінде ядролық материяның қанықтылық қасиетін дәлірек анықтауға болады деген қорытынды жасалды. $^{15}N + ^{27}Al$ ядролық жүйенің төмен энергиядағы серпімді шашырау реакциясы материалдар мен ядролық отындардың қасиеттерін зерттеу үшін маңызды. Бұл ядролық процесін зерттеу жұмысы болашақ ядролық материалдар мен энергетикалық технологияның дамуына үлес қосады.

*Кілт сөздер:* микроскопиялық талдау, серпімді шашырау, екілік фолдинг моделі, B3Y-Fetal потенциалы, нуклондар тығыздығының таралуы

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## Введение нового потенциала B3Y-Fetal при полукомикроскопическом анализе ядерной системы $^{15}N + ^{27}Al$

В экспериментальном анализе процесса упругого рассеяния на ядерной системе $^{15}N + ^{27}Al$ при низких энергиях был применен новый микроскопический метод. В микроскопическом анализе использовались новые потенциалы B3Y-Fetal, рассчитанные в вариационном методе с ограничениями низшего порядка в двухтелевой матрице (LOCV). На основе модели двойного фолдинга (МДФ) были построены микрофолдинг потенциалы CDM3Y2-Paris, CDB3Y2-Fetal и CDB3Y3-Fetal. Эти потенциалы были получены путем настройки плотностных параметров $C$, $\alpha$, $\beta$, $\gamma$. Зависимые от плотности параметры вводились с учетом эффективного взаимодействия нуклон-нуклонность (NN) через форм-фактор распределения плотности нуклонов в столкнувшихся ядрах $^{15}N$ и $^{27}Al$. Особенностью исследования является расчет введенных плотностных параметров при оптимальном значении сжимаемости — $K$, описывающем насыщение ядерного вещества. Результаты анализа оптической модели (ОМ) и МДФ показали оптимальные значения параметров оптических и фолдинг потенциалов. Теоретические сечения упругого рассеяния, полученные в феноменологическом и микроскопическом анализах, хорошо согласуются с экспериментальными данными. В полумикроскопическом анализе была определена эффективность фолдинг потенциала B3Y-Fetal. На основе анализа экспериментальных данных упругого рассеяния можно сделать вывод о возможности более точного определения насыщающих свойств ядерной материи. Упругое рассеяние на ядерной системе $^{15}N+^{27}Al$ при низких энергиях важно для исследования свойств материалов и ядерного топлива. Это исследование ядерного процесса способствует дальнейшему развитию ядерных материалов и энергетических технологий.

*Ключевые слова*: микроскопический анализ, упругое рассеяние, модель двойного фолдинга, потенциал B3Y-Fetal, распределение плотности нуклонов

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