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P.Yu. Tsyba, G.S. Altayeva, O.V. Razina*

L.N. Gumilyov Eurasian National University, Astana, Kazakhstan
*(*E-mail: olvikraz@mail.ru)*

Study of the scalar-fermionic model containing linear lagrangian fields of matter within the framework of minimal coupling

In this article, we study a model of the universe with the scalar field and the fermionic field interacting via a Yukawa-type potential. In the model, the component contributions of each of the fields are determined to the total density and pressure of dark energy. We have considered the solution of the cosmological model for the scale factor with two functional time dependences. The Yukawa-type field does not give its input to the general pressure. In the power law case, there is a significant contribution to the total increase in pressure, to the exponential — the scalar field. There are many cases when the universe makes a transition between successive epochs in various models of cosmological expansion. These regularities impose some restrictions on the profile of the scalar- fermionic field interaction and the general cosmological dynamics. Energy conditions were found to check the model under the study. In the studied model, the null energy condition, the strong energy condition, the dominant energy condition are fulfilled, and the weak energy condition, which is not mandatory, is not fulfilled. It is shown how it is possible to connect the cosmographic parameters — parameters of deceleration q, jerk j and snap s with the power low the scale factor. We investigated these restrictions using cosmological solutions with an evolving equation of state, such that a smooth transition between different epochs can always occur in the universe. The scalar-fermionic model under study describes the accelerated modes of expansion of the universe. The obtained solutions correspond to the results predicted by the theory and observational data.

Keywords: scalar field, fermionic field, Yukawa-type interaction, minimal coupling, energy condition, the cosmography.

Introduction

It is known that the universe is expanding with an acceleration, which is confirmed by the observational data. One of the simplest candidates for the description of the dark energy, which causes acceleration, is the cosmological constant Λ, introduced by Einstein in the theory of GR to describe the static universe. Space-time with a positive cosmological constant is known as the de Sitter space-time. Therefore, the universe can be described by the de Sitter geometry. The fermionic fields in cosmological models can be responsible for accelerated periods in the evolution of the universe. In most of these models, the fermionic field plays the role of inflation in the early universe and the role of dark energy in the later universe. These fermionic fields have been studied using several approaches, the results of which include numerical solutions, exact solutions, scenarios from anisotropy to isotropy, and cyclic cosmology [1-3]. The fermionic fields can be combined with other components, such as the canonical and non-canonical scalar fields. In particular, inflation and dark energy can be modeled in several ways. One subclass of these models considers this component as the tachyon field, and this idea goes back to the models of string theory. Compared to the usual canonical the scalar field, the resulting physics becomes richer as a whole, due to nonlinear effects. Depending on the initial conditions and interaction with other sources, these tachyons could contribute to the accelerated period of the later universe and be considered as a contribution of the dark energy. They can be permanent even in the early universe, where they can be associated with the inflationary period. In works [4-13] the scalar and fermionic fields were considered responsible both for the inflationary period and for the modern accelerated expansion. This effect is recognized as a consequence of the dynamics of the model itself. The exotic nature of these components does not contradict the observational data, and an important point of discussion is their comparison with the canonical scalar field [14-18].

The work is structured as follows: we construct a model through an action, which represents the sum of the Lagrangian densities of gravitational, the scalar fields, the fermionic fields and Yukawa-type fields. The field equations of Dirac, Einstein, and Klein-Gordon are derived from the variation of the field action [19-21].

The dependence of the scale factor on time reflects the main events in the history of the universe. Moreover, it is the deceleration parameter that dictates the rate of the expansion of the Hubble sphere and determines the dynamic change in the number of observed galaxies: depending on the sign of the deceleration parameter, this number either increases (in the case of slow expansion), or we remain completely alone in space (if the expansion is accelerated).

When checking fermionic sources as responsible for the accelerated expansion of the universe, different regimes appear. In the scenario of the early universe, the fermionic field grows rapidly and the matter is created until it begins to dominate and as a result, the initial accelerated expansion slows down. When the universe enters the domain of matter dominance, then the fermionic field again dominates, which leads to an era of accelerated growth rates of the scale factor. In this case, the fermionic field is responsible for inflation in the early universe and dark energy for the later universe, without the need for a cosmological study of constant terms or a scalar field. In the later universe, energy again begins to dominate and there is a gradual transition to dark energy, the so-called fermionic energy period, in which the accelerated regime begins and continues into the modern era [22-33].

Methods

Let us consider the general action in the form

$$S=\int\sqrt{-g}d^4x\left\{\frac{R}{2}+\frac{1}{2}\partial^\mu\phi\partial_\mu\phi-V_1(\phi)+\frac{i}{2}\left[\bar{\psi}\Gamma^\mu D_\mu\psi-(D_\mu\bar{\psi})\Gamma^\mu\psi\right]-V_2(\bar{\psi}\psi)-\lambda\bar{\psi}\phi\psi\right\}. \tag{1}$$

Here the sources of gravity are: the density of the Lagrangian of the scalar field ϕ

$$L_b=\frac{1}{2}\partial^\mu\phi\partial_\mu\phi-V_1(\phi),$$

where $V_1(\phi)$ is the scalar field self-interaction potential.

The fermionic field with the potential of self-interaction $V_2(\bar{\psi}\psi)$

$$L_f=\frac{i}{2}\left[\bar{\psi}\Gamma^\mu D_\mu\psi-(D_\mu\bar{\psi})\Gamma^\mu\psi\right]-V_2(\bar{\psi}\psi),$$

where ψ and $\bar{\psi}=\psi^\dagger\gamma^0$ represent the spinor field and its adjoint, respectively.

The Lagrangian density corresponds to the Yukawa type interaction showing the relationship between the scalar and the fermionic fields

$$L_Y=-\lambda\bar{\psi}\phi\psi,$$

where λ is the coupling constant of the Yukawa potential.

From the variation of the action (1) on the spinor field ψ and its conjugate $\bar{\psi}$ follow the Dirac equations

$$i\Gamma^\mu D_\mu\psi-\frac{\partial V_2(\bar{\psi}\psi)}{\partial\bar{\psi}}-\lambda\phi\psi=0,$$

$$iD_\mu\bar{\psi}\Gamma^\mu+\frac{\partial V_2(\bar{\psi}\psi)}{\partial\psi}+\lambda\bar{\psi}\phi=0,$$

in addition, variation of action (1) with respect to the scalar field ϕ gives the Klein-Gordon equation for the scalar field, which has the form

$$\nabla_\mu \nabla^\mu \phi + V_1(\phi) + \lambda\bar{\psi}\psi = 0.$$

The Einstein equations are obtained by varying the action (1) with respect to the metric tensor

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -T_{\mu\nu},$$

where $T_{\mu\nu}$ the energy-momentum tensor which is equal to the following expression

$$T^{\mu\nu} = \frac{i}{4}\left[\bar{\psi}\Gamma^\mu D^\nu \psi - D^\mu \bar{\psi}\Gamma^\nu \psi\right] + \partial^\mu \phi \partial^\nu \phi -$$

$$-g^{\mu\nu}\left[\frac{1}{2}\partial^\sigma \phi \partial_\sigma \phi - V_1(\phi) - \lambda\bar{\psi}\phi\psi + \frac{i}{2}(\bar{\psi}\Gamma^\lambda D_\lambda \psi - D_\lambda \bar{\psi}\Gamma^\lambda \psi) - V_2(\bar{\psi}\psi)\right].$$

In order to study the evolution of a homogeneous and isotropic spatially flat universe, we use the Friedmann-Robertson-Walker metric

$$ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2), \qquad (2)$$

where $a(t)$ is the scale factor of the universe. In this metric, the components of the tetrad are given by the formula as $e_0^\mu = \delta_0^\mu$ and $e_i^\mu = \delta_i^\mu / a(t)$, Dirac matrices and spin connectivity are equal

$$\Gamma^0 = \gamma^0,$$

$$\Gamma^0 = \frac{1}{a(t)}\gamma^i,$$

$$\Omega_0 = 0,$$

$$\Omega_i = \frac{1}{2}\dot{a}(t)\gamma^i \gamma_0.$$

Here, the dot means the time derivative, and in what follows we will use the bilinear function $u = \bar{\psi}\psi$ and let us introduce the notation $V_{2u} = \frac{dV_2}{du}$. Then the Lagrangian of the model under study (1) will take the form

$$L = -3a\dot{a}^2 + a^3\frac{1}{2}\dot{\phi}^2 - a^3 V_1(\phi) + a^3\frac{i}{2}(\bar{\psi}\gamma^0\dot{\psi} - \dot{\bar{\psi}}\gamma^0\psi) - a^3 V_2(u) - a^3\lambda u\phi. \qquad (3)$$

From the Euler-Lagrange and energy-momentum tenzor, the complete system of equations of motion corresponding to the Lagrangian (3) is

$$3H^2 = \rho, \qquad (4)$$

$$2\dot{H} + 3H^2 = -p, \qquad (5)$$

$$\dot{\psi} + \frac{3}{2}H\psi + i\gamma^0 V_{2u}\psi + i\lambda\gamma^0\psi\phi = 0, \qquad (6)$$

$$\dot{\bar{\psi}} + \frac{3}{2}H\bar{\psi} - i\gamma^0 V_{2u}\bar{\psi} - i\lambda\phi\bar{\psi}\gamma^0 = 0, \qquad (7)$$

$$\ddot{\phi} + 3H\dot{\phi} + V_{1\phi} + \lambda u = 0, \qquad (8)$$

where

$$\rho = \frac{\dot{\phi}^2}{2} + V_1(\phi) + V_2(u) + \lambda u\phi, \qquad (9)$$

$$p = \frac{\dot{\phi}^2}{2} - V_1(\phi) + V_{2u}(u) - V_2(u). \qquad (10)$$

For further calculations, we need an expression that follows from the definition of the bilinear function u and equations (6)-(7), valid for an arbitrary potential V

$$u = \frac{c}{a^3},$$

where c is the constant of integration.

Results

Example 1

The system of equations (4)-(10) can have the following solution in the form of an exponential function of the scale factor and the scalar field

$$a = a_0 e^{\alpha t}, \qquad (11)$$

$$\phi = \phi_0 e^{-\gamma t},$$

where a_0,ϕ_0, α and γ are some constants and $\alpha > 0$, $\gamma > 0$. Then from the Klein-Gordon equation (8) the potential of the scalar field V_1 is equal

$$V_1 = -\frac{1}{2}\phi_0^2\gamma^2 e^{-2\gamma t} + \frac{3}{2}\alpha\phi_0^2\gamma e^{-2\gamma t} - \frac{\lambda\phi_0 c\gamma e^{-(3\alpha+\gamma)t}}{a_0^3(3\alpha+\gamma)} + V_{10},$$

where V_{10} is the constant of integration. From equations (8) and (9) we find the potential of the fermionic field V_2

$$V_2 = -3\alpha\phi_0\gamma\left(\frac{1}{2}\phi_0 e^{-2\gamma t} + \frac{\lambda c e^{-(3\alpha+\gamma)t}}{a_0^3(3\alpha+\gamma)}\right) + V_{20}.$$

The total energy density and pressure of the model under study from the Friedman equations (4) and (5) are

$$\rho = 3\alpha^2,$$

$$p = -3\alpha^2,$$

where the conditions $3\alpha^2 = V_{10} + V_{20}$ are satisfied for the constants.

The componentwise contributions of each of the fields to the total density (9), respectively, are equal to

$$\rho_b = \frac{\dot{\phi}^2}{2} + V_1(\phi) = \frac{3}{2}\alpha\phi_0^2\gamma e^{-2\gamma t} - \frac{\lambda\phi_0\gamma c e^{-(3\alpha+\gamma)t}}{a_0^3(3\alpha+\gamma)} + 3\alpha^2 - V_{20},$$

$$\rho_f = V_2(u) = -3\alpha\phi_0\gamma\left(\frac{1}{2}\phi_0 e^{-2\gamma t} + \frac{\lambda c e^{-(3\alpha+\gamma)t}}{a_0^3(3\alpha+\gamma)}\right) + V_{20},$$

$$\rho_Y = \lambda u\phi = \frac{\lambda c\phi_0}{a_0^3} e^{-(3\alpha+\gamma)t}.$$

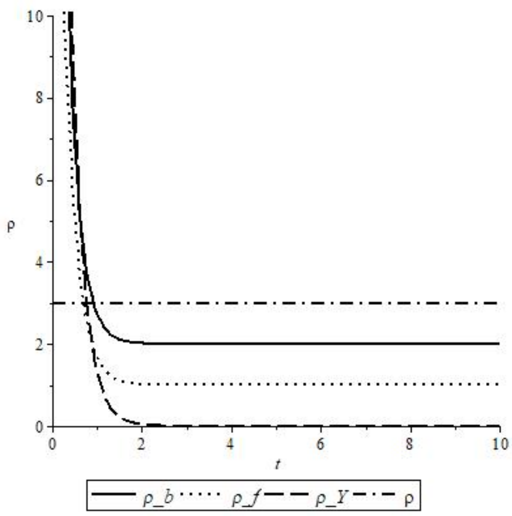


Figure 1. The energy density ρ and the componentwise contributions of the densities versus time t, at $\phi_0 = 4, c = 2, a_0 = 1, \alpha = 1, \gamma = 2, \lambda = 6, V_{20} = 1$.

Figure 1 shows the scalar field ρ_b (solid line), the fermionic field density ρ_f (dotted line), Yukawa type field density ρ_Y (dash line), the total dark energy density ρ (dash-dotted line). The figure shows that the componentwise contributions of the densities in the early epoch change with time, while the total density of dark energy remains constant.

The componentwise contributions of each of the fields to the total pressure (10) are respectively equal to

$$p_b = \frac{\dot{\phi}^2}{2} - V_1(\phi) = \phi_0^2\gamma^2 e^{-2\gamma t} - \frac{3}{2}\phi_0^2\gamma e^{-2\gamma t} + \frac{\lambda\phi_0\gamma c e^{-(3\alpha+\gamma)t}}{a_0^3(3\alpha+\gamma)} - 3\alpha^2 + V_{20},$$

$$p_f = V_{2u}(u) - V_2(u) - \phi_0^2\gamma^2 e^{-2\gamma t} - \frac{\lambda c\phi_0}{a_0^3} e^{-(3\alpha+\gamma)t} + 3\alpha\left[\frac{1}{2}\phi_0^2\gamma e^{-2\gamma t} + \frac{\lambda\phi_0\gamma c e^{-(3\alpha+\gamma)t}}{a_0^3(3\alpha+\gamma)}\right] - V_{20}.$$

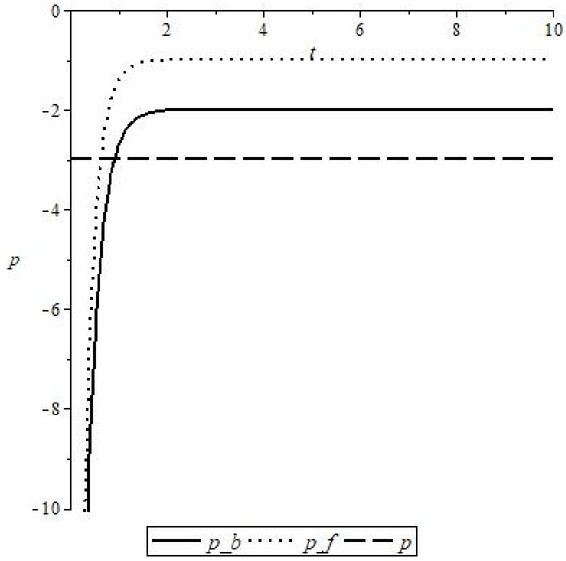


Figure 2. Total pressure p and componentwise contributions of the pressures versus time t, at $\phi_0 = 4, c = 2, a_0 = 1, \alpha = 1, \gamma = 2, \lambda = 6, V_{20} = 1$.

Figure 2 shows the scalar field pressure p_s (solid line), the fermionic field pressure p_f (dotted line), the total pressure p (dash line). The Yukawa-type field does not contribute to the total pressure. As seen from Figure 2, the scalar field has a greater effect on the total pressure.

The solution for the fermionic field will be sought in the form

$$\psi_k(t) = A_k(t)e^{iD_k(t)}, k = 0,1,2,3, \tag{12}$$

where $A_k(t)$ and $D_k(t)$ in the arbitrary functions of time.

Let us substitute the general form of the fermionic field function (12) into the Dirac equation (6), as a result we obtain

$$\psi_0(t) = A_{00}e^{-\frac{3}{2}\alpha t}\exp\left[i\left(\frac{\phi_0^2\gamma^2 a_0^3}{c}\frac{1}{3\alpha-2\gamma}e^{(3\alpha-2\gamma)t} + D_{00}\right)\right],$$

$$\psi_1(t) = A_{01}e^{-\frac{3}{2}\alpha t}\exp\left[i\left(\frac{\phi_0^2\gamma^2 a_0^3}{c}\frac{1}{3\alpha-2\gamma}e^{(3\alpha-2\gamma)t} + D_{01}\right)\right],$$

$$\psi_2(t) = A_{02}e^{-\frac{3}{2}\alpha t}\exp\left[i\left(-\frac{\phi_0^2\gamma^2 a_0^3}{c}\frac{1}{3\alpha-2\gamma}e^{(3\alpha-2\gamma)t} + D_{02}\right)\right],$$

$$\psi_3(t) = A_{03}e^{-\frac{3}{2}\alpha t}\exp\left[i\left(-\frac{\phi_0^2\gamma^2 a_0^3}{c}\frac{1}{3\alpha-2\gamma}e^{(3\alpha-2\gamma)t} + D_{03}\right)\right],$$

where A_{0k} and D_{0k} at $k = 0,1,2,3$ are the integration constants. Conjugated field $\bar{\psi} = \psi^\dagger\gamma^0$ respectively equal to

$$\bar{\psi}_0(t) = A_{00}e^{-\frac{3}{2}\alpha t}\exp\left[-i\left(\frac{\phi_0^2\gamma^2a_0^3}{c}\frac{1}{3\alpha-2\gamma}e^{(3\alpha-2\gamma)t}+D_{00}\right)\right],$$
$$\bar{\psi}_1(t) = A_{01}e^{-\frac{3}{2}\alpha t}\exp\left[-i\left(\frac{\phi_0^2\gamma^2a_0^3}{c}\frac{1}{3\alpha-2\gamma}e^{(3\alpha-2\gamma)t}+D_{01}\right)\right],$$

$$\bar{\psi}_2(t) = -A_{02}e^{-\frac{3}{2}\alpha t}\exp\left[-i\left(-\frac{\phi_0^2\gamma^2a_0^3}{c}\frac{1}{3\alpha-2\gamma}e^{(3\alpha-2\gamma)t}+D_{02}\right)\right],$$
$$\bar{\psi}_3(t) = -A_{03}e^{-\frac{3}{2}\alpha t}\exp\left[-i\left(-\frac{\phi_0^2\gamma^2a_0^3}{c}\frac{1}{3\alpha-2\gamma}e^{(3\alpha-2\gamma)t}+D_{03}\right)\right],$$

where the conditions for the constants are satisfied $\frac{c}{a_0^3} = A_{00}^2 + A_{01}^2 - A_{02}^2 - A_{03}^2$, the definition of a bilinear function that follows from $u = \bar{\psi}\psi = \frac{c}{a_0^3}$.

Example 2

The system of equations (4)-(10) can have the following solution in the form of the power low the scale factor and the scalar field

$$a = a_0t^{\alpha}, \tag{13}$$

$$\phi = \phi_0t^{\gamma},$$

where a_0,ϕ_0, α and γ are constants and $\alpha > 1$, $\gamma < 0$. Then from the Klein-Gordon equation (8) the potential of the scalar field V_1 is

$$V_1 = -\frac{1}{2}\phi_0^2\gamma^2t^{2(\gamma-1)} - \frac{3\alpha\phi_0^2\gamma^2t^{2(\gamma-1)}}{2(\gamma-1)} - \frac{\lambda\phi_0c\gamma t^{(\gamma-3\alpha)}}{a_0^3(\gamma-3\alpha)} + V_{10},$$

where V_{10} is the integration constant. From equations (9) and (10) we find the potential of the fermionic field V_2

$$V_2 = 3\alpha\left(\frac{\phi_0^2\gamma^2t^{2(\gamma-1)}}{2(\gamma-1)} + \frac{\alpha}{t^2} - \frac{\lambda\phi_0ct^{\gamma-3\alpha}}{a_0^3(\gamma-3\alpha)}\right) + V_{20}.$$

The total energy density and pressure of the model under study from the Friedman equations (4) and (5) are

$$\rho = \frac{3\alpha^2}{t^2}, \tag{14}$$

$$p = \frac{\alpha}{t^2}(2-3\alpha),$$

where the conditions $V_{10} + V_{20} = 0$ are satisfied for the constants.

The componentwise contributions of each of the fields to the total density (14) are respectively equal to

$$\rho_b = \frac{\dot{\phi}^2}{2} + V_1(\phi) = -\frac{3\alpha\phi_0^2\gamma t^{2(\gamma-1)}}{2(\gamma-1)} - \frac{\lambda\phi_0\gamma ct^{(\gamma-3\alpha)}}{a_0^3(\gamma-3\alpha)} + V_{10},$$

$$\rho_f = V_2(u) = \frac{3\alpha\phi_0^2\gamma^2t^{2(\gamma-1)}}{2(\gamma-1)} + \frac{3\alpha^2}{t^2} - \frac{3\alpha\lambda\phi_0ct^{(\gamma-3\alpha)}}{a_0^3(\gamma-3\alpha)} - V_{10},$$

$$\rho_Y = \lambda u\phi = \frac{\lambda c\phi_0t^{\gamma-3\alpha}}{a_0^3}.$$

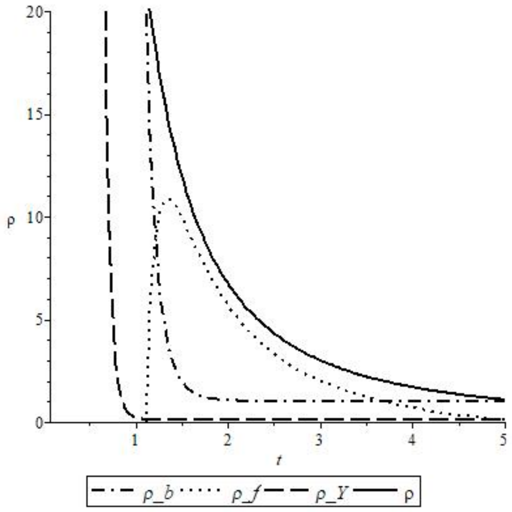


Figure 3. The energy density ρ and the componentwise contributions of the densities versus time t, at $V_{10}=0, \phi_0=1, c=1, a_0=2, \alpha=3, \gamma=-2, \lambda=0.5, a_0=2$.

Figure 3 shows the scalar field density ρ_b (dash-dotted line), the fermionic field density ρ_f (dotted line), Yukawa-type field density ρ_Y (dash line), the total dark energy density ρ (solid line).

The componentwise contributions of each of the fields to the total pressure (10) are respectively equal to

$$p_b=\frac{\dot{\phi}^2}{2}-V_1(\phi)=\phi_0^2\gamma^2t^{2(\gamma-1)}+\frac{3\alpha\phi_0^2\gamma^2t^{2(\gamma-1)}}{2(\gamma-1)}-\frac{\lambda\phi_0\gamma ct^{(\gamma-3\alpha)}}{a_0^3(\gamma-3\alpha)}-V_{10},$$

$$p_f=V_{2u}(u)-V_2(u)-\phi_0^2\gamma^2t^{2(\gamma-1)}+\frac{2\alpha}{t^2}-\frac{\lambda\phi_0t^\gamma c}{a_0^3t^{3\alpha}}-\frac{3\alpha\phi_0^2\gamma^2t^{2(\gamma-1)}}{2(\gamma-1)}-\frac{3\alpha^2}{t^2}+\frac{3\alpha\lambda\phi_0ct^{(\gamma-3\alpha)}}{a_0^3(\gamma-3\alpha)}+V_{10}.$$

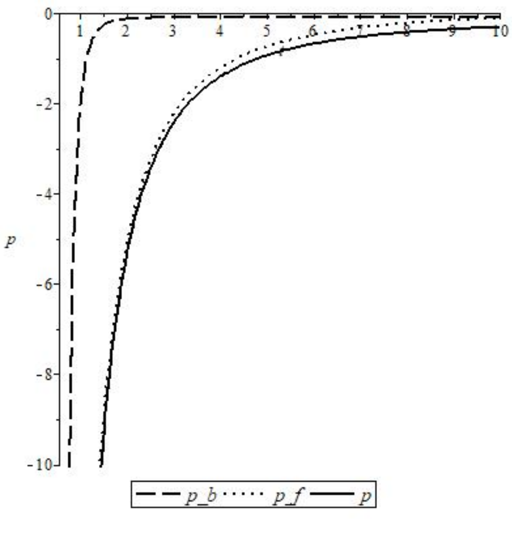


Figure 4. Total pressure p and componentwise contributions of the pressures versus time t, at $V_{10}=0, \phi_0=1, c=1, a_0=2, \alpha=2, \gamma=-2, \lambda=0.5$.

Figure 4 shows the scalar field pressure p_b (dash line), the fermionic field pressure p_f (dotted line), the total pressure p (solid line). In this case, the fermionic field contributes more to the total pressure.

The solution for the fermionic field will be sought in the form (12). We substitute the general form of the fermionic field function (12) into the Dirac equation (6), as a result, we obtain

$$\psi_0(t)=\frac{A_{00}}{t^{\frac{3}{2}\alpha}}\exp\left[i\left(\frac{\phi_0^2\gamma^2a_0^3t^{(3\alpha+2\gamma)}}{c(3\alpha+2\gamma)}-\frac{2\alpha a_0^3t^{3\alpha-1}}{c(3\alpha-1)}+D_{00}\right)\right],$$

$$\psi_1(t)=\frac{A_{01}}{t^{\frac{3}{2}\alpha}}\exp\left[i\left(\frac{\phi_0^2\gamma^2a_0^3t^{(3\alpha+2\gamma)}}{c(3\alpha+2\gamma)}-\frac{2\alpha a_0^3t^{3\alpha-1}}{c(3\alpha-1)}+D_{01}\right)\right],$$

$$\psi_2(t)=\frac{A_{02}}{t^{\frac{3}{2}\alpha}}\exp\left[i\left(-\frac{\phi_0^2\gamma^2a_0^3t^{(3\alpha+2\gamma)}}{c(3\alpha+2\gamma)}+\frac{2\alpha a_0^3t^{3\alpha-1}}{c(3\alpha-1)}+D_{02}\right)\right],$$

$$\psi_3(t)=\frac{A_{03}}{t^{\frac{3}{2}\alpha}}\exp\left[i\left(-\frac{\phi_0^2\gamma^2a_0^3t^{(3\alpha+2\gamma)}}{c(3\alpha+2\gamma)}+\frac{2\alpha a_0^3t^{3\alpha-1}}{c(3\alpha-1)}+D_{03}\right)\right],$$

where A_{0k} and D_{0k} at $k=0,1,2,3$ are the integration constants. Conjugated field $\bar{\psi}=\psi^{\dagger}\gamma^0$ respectively equal to

$$\bar{\psi}_0(t)=\frac{A_{00}}{t^{\frac{3}{2}\alpha}}\exp\left[-i\left(\frac{\phi_0^2\gamma^2a_0^3t^{(3\alpha+2\gamma)}}{c(3\alpha+2\gamma)}-\frac{2\alpha a_0^3t^{3\alpha-1}}{c(3\alpha-1)}+D_{00}\right)\right],$$

$$\bar{\psi}_1(t)=\frac{A_{01}}{t^{\frac{3}{2}\alpha}}\exp\left[-i\left(\frac{\phi_0^2\gamma^2a_0^3t^{(3\alpha+2\gamma)}}{c(3\alpha+2\gamma)}-\frac{2\alpha a_0^3t^{3\alpha-1}}{c(3\alpha-1)}+D_{01}\right)\right],$$

$$\bar{\psi}_2(t)=-\frac{A_{02}}{t^{\frac{3}{2}\alpha}}\exp\left[-i\left(-\frac{\phi_0^2\gamma^2a_0^3t^{(3\alpha+2\gamma)}}{c(3\alpha+2\gamma)}+\frac{2\alpha a_0^3t^{3\alpha-1}}{c(3\alpha-1)}+D_{02}\right)\right],$$

$$\bar{\psi}_3(t)=-\frac{A_{03}}{t^{\frac{3}{2}\alpha}}\exp\left[-i\left(-\frac{\phi_0^2\gamma^2a_0^3t^{(3\alpha+2\gamma)}}{c(3\alpha+2\gamma)}+\frac{2\alpha a_0^3t^{3\alpha-1}}{c(3\alpha-1)}+D_{03}\right)\right].$$

Cosmography

Cosmography makes it possible to test cosmological models that do not contradict the cosmological principle [33]. The components of dark energy introduced by us in the model change the equations of motion, but they do not affect the relationship between kinematic characteristics. Another approach to finding different models of dark energy is to use a pair of state determinants (j,s). It is known that the rate of expansion of the universe can be expressed through the scale factor, and the deceleration parameter q corresponds to the second derivative of the scale factor. The jerk parameter j corresponds to the fourth derivative of the scale factor, and the third snap parameter (s). Expansion of the scale factor in a Taylor series in the vicinity of the current moment of time t_0 leads to an expression that depends only on the metric (2) and is completely independent of the model [34].

$$a(t)=a_0+\dot{a}(t_0)+\tfrac{1}{2!}\ddot{a}(t_0)(t-t_0)^2+\tfrac{1}{3!}\dddot{a}(t_0)(t-t_0)^3+\tfrac{1}{4!}\ddot{a}(t_0)(t-t_0)^4, \quad (15)$$

where 0 means the current value of the quantity and terms above the fifth order have been omitted. The sign of j determines the change of the universe's dynamics, a positive value indicating the occurrence of a transition time during which the universe modifies its expansion. Moreover, the value of s is necessary to discriminate between an evolving dark energy term or a cosmological constant behaviour.

Let us represent relation (15) in the form

$$\tfrac{a(t)}{a(t_0)}=1+H_0(t-t_0)-\tfrac{q_0}{2}H_0^2(t-t_0)^2+\ldots,$$

where deceleration parameter

$$q(t) = -\frac{\ddot{a}(t)a(t)}{\dot{a}^2(t)} = -\frac{\ddot{a}(t)}{a(t)}\frac{1}{H^2(t)},$$

where a is the scale factor of the universe, and the dots denote derivatives with respect to proper time. The expansion of the universe is considered to be accelerated if $\ddot{a} > 0$, in which case the deceleration parameter will be negative.

For the most complete description of the kinematics of the cosmological expansion, it is useful to consider the expanded set of parameters. The function in terms of derivatives of the scale factor and their value at the power low the scale factor (13) is equal to:

Hubble parameter

$$H(t) = \frac{1}{a}\frac{da}{dt} = \frac{\alpha}{t},$$

Deceleration parameter

$$q(t) = -\frac{1}{a}\frac{d^2a}{dt^2}\left(\frac{1}{a}\frac{da}{dt}\right)^{-2} = -1 + \frac{1}{\alpha},$$

Jerk parameter

$$j(t) = \frac{1}{a}\frac{d^3a}{dt^3}\left(\frac{1}{a}\frac{da}{dt}\right)^{-3} = 1 - \frac{3}{\alpha} + \frac{2}{\alpha^2},$$

Snap parameter

$$s(t) = \frac{1}{a}\frac{d^4a}{dt^4}\left(\frac{1}{a}\frac{da}{dt}\right)^{-4} = 1 - \frac{6}{\alpha} + \frac{11}{\alpha^2} - \frac{6}{\alpha^3}.$$

The parameters of deceleration, jerk and snap are dimensionless. These parameters are used to study the dynamics of the later universe. The physical properties of the coefficients can be deduced in the form of the Hubble expansion. In particular, the sign of the parameter q indicates whether the Universe is accelerating or decelerating. The positive sign j defines a change in dynamics, indicating the emergence of a transitional time during which the universe modifies its expansion [35]. The meaning s is necessary to distinguish the evolutionary term of dark energy or the behavior of the cosmological constant. Using them, you can rewrite expression (15) as

$$a(t) = a_0\left[1 + H_0(t - t_0) - \frac{1}{2!}q_0H_0^2(t - t_0)^2 + \frac{1}{3!}j_0H_0^3(t - t_0)^3 + \frac{1}{4!}s_0H_0^4(t - t_0)^4\right].$$

Accelerated growth of the scale factor occurs at $q < 0$. Accelerated increase in the speed of expansion, $H > 0$, corresponds to $q < -1$.

Energy conditions

In the GR theory and modified theories of gravity, the distribution of mass, the momentum and the angular momentum must have values for any field described by the energy-momentum tensor or the matter tensor. However, Einstein's field equation does not impose restrictions on the types of states of matter or non-gravitational regions allowed in the space-time model. This is a strong point, since general relativity should be as independent as possible from any assumptions of non-gravitational physics. The weak point is that Einstein's equation allows for solutions based on properties that most cosmologists regard as non-physical, i.e. too unusual to correspond to anything in the real universe. The energy conditions are such criteria. They describe properties characteristic of all states of matter and all non-gravitational regions studied in physics. Many non-physical solutions of Einstein's equations can be excluded with the help of energy conditions. In cosmology, these four energy conditions are of great importance.

Null Energy Condition (NEC)

$$\rho + p \geq 0.$$

Weak Energy Condition (WEC)

$$\rho \geq 0, \rho + p \geq 0.$$

Strong Energy Condition (SEC)

$$\rho + 3p \geq 0, \rho + p \geq 0.$$

Dominant Energy Condition (DEC)

$$\rho \geq 0, -\rho \leq p \leq \rho.$$

For our model (13) the energy conditions will take the following form

NEC

$\frac{2\alpha}{t^2} \geq 0.$

WEC

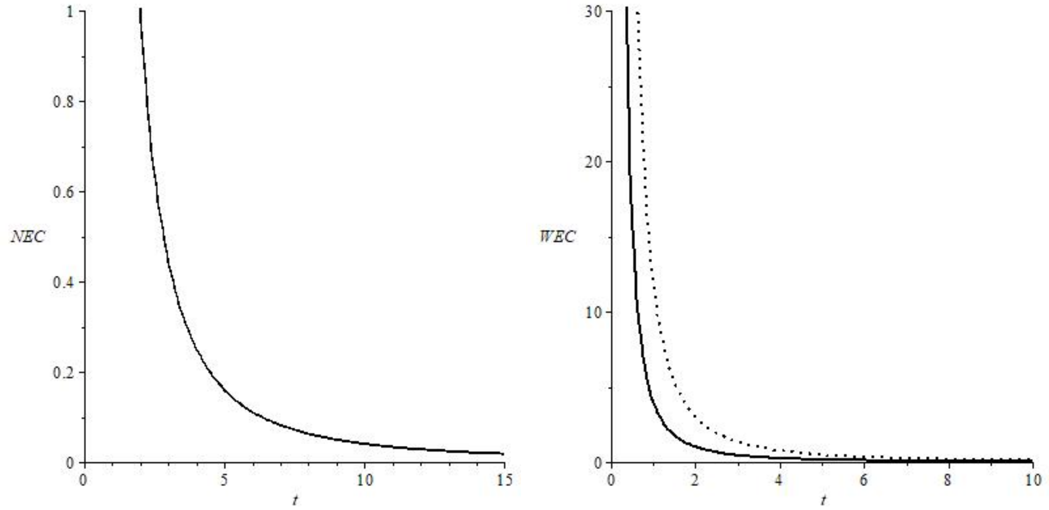
$$\frac{3\alpha^2}{t^2} \geq 0, \quad \frac{2\alpha}{t^2} \geq 0.$$

SEC

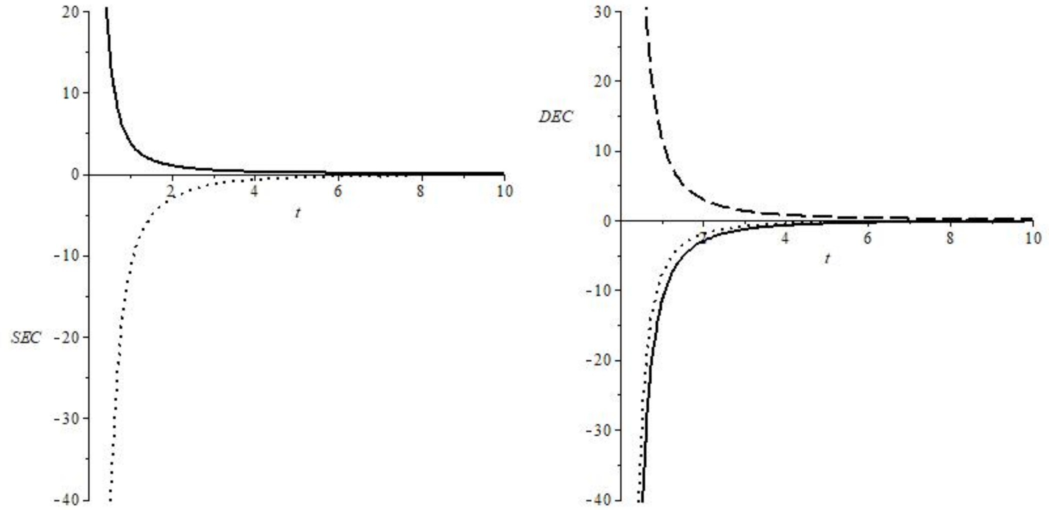
$$\frac{6\alpha}{t^2}(1-\alpha) \geq 0, \quad \frac{2\alpha}{t^2} \geq 0.$$

DEC

$$\frac{3\alpha^2}{t^2} \geq 0, \qquad -\frac{3\alpha^2}{t^2} \leq \frac{(2-3\alpha)\alpha}{t^2} \leq \frac{3\alpha^2}{t^2}.$$



(a) Null energy Condition (5) (b) Strong energy Condition (5)



(c) Weak energy Condition (5) (d) Dominant energy Condition (5)

Figure 5. Energy Condition

NEC is shown in Figure 5(a) ($\rho + p$ is a solid line). WEC is shown in Figure 5(b) (ρ — solid line, $\rho + p$ — dotted line). SEC is shown in Figure 5(c) ($\rho + 3p$ — dotted line, $\rho + p$ — solid line). DEC is shown in Figure 5(d) (ρ — dashed line, ρ — solid line, p — dotted line). These conditions impose simple and model-independent constraints on the behavior of energy density and pressure. For our model, a null energy condition, a strong energy condition, a dominant energy condition, and a weak energy condition, which is not mandatory, are fulfilled.

Conclusion

In this work, we analyzed a cosmological model with two components, the scalar field and the fermionic field, interacting through a Yukawa-type potential. Cosmological solutions are obtained analytically for a given value of the exponential and power scale factor. We investigate a particular solution in which it was found that the Yukawa-type potential affects the density but does not contribute to the total pressure. These limitations were studied using cosmological solutions with evolving equations of state, such that a smooth transition between different epochs can always occur in the universe. We also considered some scalar-fermionic model that can describe a single of dark energy — dark matter. To describe the kinematics of the cosmological expansion, we found a wide set of parameters a small power scaling factor: the deceleration parameter q, the jerk parameter j, and the snap parameter s.

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References

1 Ribas M.O Cosmological model with fermion and tachyon fields interacting via Yukawa-type potential / M.O. Ribas, F.P. Devecchu, G.M. Kremer / M.O. Ribas, F.P. Devecchu, G.M. Kremer // Modern Physics Letters A. — 2016. — Vol. 1. — No. 9. — P. 1650039.

2 Riess A.G. Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant / A.G. Riess., A.V. Filippenko, P. Challis, et al. // Astrophysical Journal. — 1998. — Vol. 116. — No. 3. — P. 1009.

3 Riess A.G. Type Ia supernova discoveries at z>1 from the Hubble Space Telescope: Evidence for past deceleration and constraints on dark energy evolution / A.G. Riess et al. // Astrophysical Journal. — 2004. — Vol. 607. — P. 665–687.

4 Ribas M.O. Fermionic cosmologies with Yukawa type interactions / M.O. Ribas, F.P. Devecchu, G.M. Kremer // Europhysics Letters Journal. — 2011. — Vol. 93. — No.2. — P.19002.

5 Nojiri S. The future evolution and finite-time singularities in F (R)-gravity unifying the inflation and cosmic acceleration / S. Nojiri, S.D. Odintsov // Physical Review D. — 2008. — Vol. 78. — P. 046006.

6 Nojiri S. Introduction to Modified Gravity and Gravitational Alternative for Dark Energy / S. Nojiri, S.D. Odintsov // International Journal of Geometric Methods in Modern Physics. — 2007. — Vol. 04. — No. 01. — P. 115–145.

7 Bamba K. Future of the universe in modified gravitational theories: Approaching to the finite-time future singularity / K. Bamba, S. Nojiri, S.D. Odintsov // Journal of Cosmology and Astroparticle Physics. — 2008. — Vol. 10. — P. 045.

8 Bamba K. Finite-time future singularities in modified Gauss-Bonnet and F (R, G) gravity and singularity avoidance / K. Bamba, S.D. Odintsov, L. Sebastiani, S. Zerbini // The European Physical Journal C. — 2010. — Vol. 67. — P. 295.

9 Padmanabhan T. Accelerated expansion of the universe driven by tachyonic matter / T. Padmanabhan // Physical Review D. — 2002. — Vol. 66. — P. 021301.

10 Abramo L.R.W. Cosmological dynamics of the tachyon with an inverse power-law potential / L.R.W. Abramo, F. Finelli // Physics Letters B. — 2003. — Vol. 575. — No. 3. — P. 165.

11 Calcagni G. Tachyon dark energy models: Dynamics and constraints / G. Calcagni, A.R. Liddle // Physical Review D. — 2006. — Vol. 74. — P. 043528.

12 Myrzakulov R. Cosmological solutions in F (T) gravity with the presence of spinor fields / R. Myrzakulov, D. Saez-Gomez, P. Tsyba // International Journal of Geometric Methods in Modern Physics. — 2015. — Vol. 12. — P. 1550023.

13 Yerzhanov K. g-Essence as the cosmic speed-up / K. Yerzhanov, K. Yesmakhanova, P. Tsyba, N. Myrzakulov, G. Nugmanova, R. Myrzakulov // Astrophysics and Space Science. — 2012. — Vol. 341, No. 2. — P. 681–688.

14 Avelino P. The politics of sustainability transitions / P. Avelino et al. // Journal of Environmental Policy and Planning. — 2016. — Vol. 8. — No. 2. — P.70.

15 Joyce A. Trodden M. Beyond the cosmological standard model / A. Joyce, B. Jain, J. Khoury // Physics Reports. — 2015. — Vol. 568. — No. 2. — P. 1–98.

16 Bamba K. Dark Energy Cosmology: The Equivalent Description via Different Theoretical Models and Cosmography Tests / K. Bamba, S. Capozziello, S. Nojiri, S. Odintsov // Astrophysics and Space Science. — 2012. — Vol. 342. — P. 155–228.

17 Copeland E.J. Dynamics of dark energy / E.J. Copeland, M. Sami, S. Tsujikawa // International Journal of Modern Physics D. — 2006. — Vol. 15. — No. 11. — P. 1753–1935.

18 Capozziello S. Classifying and avoiding singularities in the alternative gravity dark energy models / S. Capozziello, M. De Laurentis, S. Nojiri, S.D. Odintsov // Physical Review D. — 2009. — Vol. 79. — P. 559.

19 Ribas M.O. Fermions as sources of accelerated regimes in cosmology / M.O. Ribas, F.P. Devecchi, G.M. Kremer // Physical Review D. — 2005. — Vol. 72. — P. 3502.

20 Samojeden L.L. Fermions in Brans-Dicke cosmology / L.L. Samojeden, F.P. Devecchi, G.M. Kremer // Physical Review D. — 2010. — Vol. 81. — P. 7301.

21 Samojeden L.L. Accelerated expansion in bosonic and fermionic 2D cosmologies with quantum effects / L.L. Samojeden, G.M. Kremer, F P. Devecchi // Europhys. Letters. — 2009. — Vol. 87. — P.1.

22 Razina O. Cosmological Einstein-Maxwell model with g-essence / O. Razina, P. Tsyba, B. Meirbekov, R. Myrzakulov // International Journal of Modern Physics D. — 2019. — Vol. 28. – No. 10. — P. 1950126.

23 Razina O. Power solution of the f (R)-gravity with Maxwell term and g-essence / O. Razina, P. Tsyba, Z.M. Sagidullayeva // Bulletin of the university of Karaganda-Physics. — 2019. — Vol. 1. — No. 93. — P. 94–102.

24 Ribas M.O. Fermions in a Walecka-type cosmology / M.O. Ribas, P. Zambianchi, F.P. Devecchi, G.M. Kremer // Europhysics Letters. — 2012. — Vol. 97. — P. 5.

25 Kulnazarov I. G-essence with Yukawa Interactions / I. Kulnazarov, K. Yerzhanov, O. Razina, Sh. Myrzakul, P. Tsyba, R. Myrzakulov // The European Physical Journal C. — 2011. — Vol. 71. — No. 7. — P. 1698.

26 Tsyba P. Scenario of the evolution of the universe with equation of state of the Weierstrass type gas / P. Tsyba, O. Razina, Z. Barkova, S. Bekov, R. Myrzakulov // Journal of Physics: Conference Series. — 2019. — Vol. 1391. — P. 012162.

27 Razina O.V. Cosmological Yang-Mills model with k-essence / O.V. Razina, P.Yu. Tsyba, R. Myrzakulov, B. Meirbekov, Z. Shanina // Journal of Physics Conference Series. — 2019. — No. 1391(1). — P. 012164.

28 Altayeva G. Cosmography in the multifield cosmological model / G. Altayeva, O. Razina, P. Tsyba // Bulletin of the University of Karaganda-Physics. — 2022. — No. 2(106). — P. 37-48.

29 Myrzakulova S.A. Slow-roll inflation in the k-essence model with a periodic scalar field function / S.A. Myrzakulova, O.V. Razina, N.A. Myrzakulov, A.B. Altaybaeva // Recent Contributions to Physics. — 2022. — Vol. 81. — No. 2. — P. 19-27.

30 Rakhatov D.Z. Power-law solutions of the f(G) gravity model with electromagnetic and scalar field / D.Z. Rakhatov, P.Y. Tsyba, O.V. Razina // Recent Contributions to Physics. — 2022. — Vol. 80. — No. 1. — P. 12-21.

31 Razina O. G-essence cosmologies with scalar-fermion interactions / O. Razina, Y. Myrzakulov, N. Serikbayev, S. Myrzakul, G. Nugmanova, R. Myrzakulov // European Physical Journal Plus. — 2011. — Vol. 126. — No. 9. — P. 85.

32 Болотин Ю.Л. Расширяющаяся Вселенная: замедление или ускорение? / Ю.Л. Болотин, Д.А. Ерохин, О.А. Лемец // Успехи физических наук. — 2012. — Т. 55. — № 9. — С. 876–918.

33 Arjona R. Novel null tests for the spatial curvature and homogeneity of the Universe and their machine learning reconstructions / R. Arjona, S. Nesseris // Physical Review D. — 2021. — Vol. 103. — P. 063537.

34 Capozziello S. Cosmography of f (R) gravity / S. Capozziello, V.F. Cardone, V. Salzano // Physical Review D. — 2008. — Vol. 78. — P. 063504.

35 Capozziello S. Gravity Cosmography / S. Capozziello, R. D'Agostino, O. Luongo // General Relativity and Quantum Cosmology. — 2019. — Vol. 1. — No. 35. — P. 82.

П.Ю. Цыба, Г.С. Алтаева, О.В. Разина

Минималды өзара әсерлесуін қамтитын материя өрістерінің сызықты лагранжиандары бар скалярлық-фермиондық модельді зерттеу

Мақалада Юкава типті потенциал арқылы әрекеттесетін скалярлық және фермиондық өрістері бар Әлемнің моделі зерттелген. Зерттелетін модельде қараңғы энергияның жалпы тығыздығы мен қысымына өрістердің әрқайсысының құрамдас бөлігі бойынша үлестері анықталған. Космологиялық модель есептерін шешуде функционалды уақытқа тәуелді екі шешімі бар масштабты фактор қарастырылған. Юкава типті өріс жалпы қысымға өз үлесін қоспайды. Дәрежелік жағдайда фермиондық өріс, ал экспоненциалды — скалярлық өрісте толық қысымға көбірек үлес қосады. Көп жағдайларда, космологиялық ұлғаюдың әртүрлі модельдерінде Әлем кезекті дәуірлер арасында ауысуды жүзеге асырады. Бұл заңдылықтар скалярлық және фермиондық өріс әсерлесу профиліне және жалпы космологиялық динамикаға белгілі бір шектеулер қояды. Зерттелетін модельді тексеру үшін энергетикалық жағдай анықталды. Зерттелетін модельде нөлдік энергетикалық жағдай, күшті энергетикалық жағдай, үстемдік энергетикалық жағдай орындалады және міндетті емес әлсіз энергетикалық жағдай орындалмайды. Космографиялық параметрлердің — q баяулату параметрі, j серпілу параметрі және s басу параметрлерінің дәрежелік масштабты факторының мәнімен қалай байланыстыруға болатындығы көрсетілген. Бұл шектеулер космологиялық шешімдерді қолдана отырып, эволюционалдық күй теңдеуімен зерттелді, осылайша Әлемде әр түрлі дәуірлер арасында бірқалыпты ауысу болуы мүмкін. Зерттеліп отырған скалярлық-фермиондық модельдер Әлемнің үдемелі ұлғаю режимдерін сипаттайды. Алынған нәтиже бойынша шешімдер, бақылау деректер нәтижелерімен теориясына болжау сәйкес келеді.

Кілт сөздер: скалярлы өріс, фермиондық өріс, Юкава типті әсерлесу, минималды әсерлесу, энергетикалық жағдай, космография.

П.Ю. Цыба, Г.С. Алтаева, О.В. Разина

Исследование скалярно-фермионной модели, содержащей линейные лагранжианы полей материи, в рамках минимального взаимодействия

В статье исследована модель Вселенной со скалярными и фермионными полями, взаимодействующими через потенциал типа Юкавы. В данной модели определены покомпонентные вклады каждого из полей в полную плотность и давление темной энергии. Рассмотрено решение космологической модели для масштабного фактора с двумя функциональными зависимостями от времени. Поле типа Юкавы не дает своего вклада в общее давление. В степенном случае больший вклад в полное давление осуществляет фермионное поле, в экспоненциальном — скалярное. Существует множество случаев, когда Вселенная совершает переход между последующими эпохами в различных моделях космологического расширения. Эти закономерности накладывают некоторые ограничения на профиль скалярно-фермионного взаимодействия и на общую космологическую динамику. Для проверки исследуемой модели найдены энергетические условия. В указанной модели выполняется нулевое энергетическое условие, сильное энергетическое условие, доминирующее энергетическое условие и не осуществляется слабое энергетическое условие, которое необязательное. Показано, как можно связать космографические параметры — параметры замедления q, рывка j и щелчка s со степенным значением масштабного фактора. Авторами исследованы ограничения, с использованием космологических решений с эволюционирующим уравнением состояния, таким, что во Вселенной всегда может произойти плавный переход между разными эпохами. В исследуемой скалярно-фермионной модели описаны ускоренные режимы расширения Вселенной. Полученные решения соответствуют результатам, предсказанным теорией.

Ключевые слова: скалярное поле, фермионное поле, взаимодействие типа Юкавы, минимальное взаимодействие, энергетические условия, космография.

References

1 Ribas, M.O., Devecchu, F.P., & Kremer, G.M. (2016). Cosmological model with fermion and tachyon fields interacting via Yukawa-type potential. *Modern Physics Letters A, 1*(9) 1650039.

2 Riess, A.G., Filippenko, A.V., & Challis, P. et al. (1998). Observational evidence from supernovae for an Accelerating Universe and a Cosmological Constant. *Astrophysical Journal, 116*(3), 1009–1038.

3 Riess, A.G., Strolger, L.G., Tonry, J., et al. (2004). Type Ia supernova discoveries at z > 1 from the hubble space telescope: evidence for past deceleration and constraints on dark energy evolution. *Astrophysical Journal, 607*, 665–687.

4 Ribas, M.O., Devecchu, F.P., & Kremer, G.M. (2011). Fermionic cosmologies with Yukawa type interactions. *Europhysics Letters Journal, 93*(2), 19002.

5 Nojiri, S., & Odintsov, S.D. (2008). Future evolution and finite-time singularities in F (R) gravity unifying inflation and cosmic acceleration. *Physical Review D, 78*, 046006.

6 Nojiri, S., & Odintsov, S.D. (2007). Introduction to modified gravity and gravitational alternative for dark energy. *International Journal of Geometric Methods in Modern Physics, 4*(1), 115–145.

7 Bamba, K., Nojiri, S., & Odintsov, S.D. (2008). Future of the universe in modified gravitational theories: Approaching to the finite-time future singularity. *Journal of Cosmology and Astroparticle Physics, 10*, 045.

8 Bamba, K., Odintsov, S.D., Sebastiani, L., et al. (2010). Finite-time future singularities in modified Gauss-Bonnet and F(R, G) gravity and singularity avoidance. *The European Physical Journal C, 67*, 295–310.

9 Padmanabhan, T. (2002). Accelerated expansion of the universe driven by tachyonic matter. *Physical Review D, 66*, 021301.

10 Abramo, L.R.W., & Finelli, F. (2003). Cosmological dynamics of the tachyon with an inverse power-law potential. *Physics Letters B, 575*(3), 165.

11 Calcagni, G., & Liddle, A.R. (2006). Tachyon dark energy models: Dynamics and constraints. *Physical Review D, 74*, 043528.

12 Myrzakulov, R., Saez-Gomez, D., & Tsyba, P. (2015). Cosmological solutions in F (T) gravity with the presence of spinor fields. *International Journal of Geometric Methods in Modern Physics, 12*, 1550023.

13 Yerzhanov, K., Yesmakhanova, K., Tsyba, P., Myrzakulov, N., Nugmanova, G., & Myrzakulov, R. (2012). g-Essence as the cosmic speed-up. *Astrophysics and Space Science, 341*(2), 681–688.

14 Avelino, P. et al. (2016). The politics of sustainability transitions. *Journal of Environmental Policy and Planning, 8*(2), 70.

15 Joyce, A., Jain, B., Khoury, J., & Trodden, M. (2015). Beyond the cosmological standard model. *Physics Reports, 568*(2), 1–98.

16 Bamba, K., Capozziello, S., Nojiri, S., & Odintsov, S. (2012). Dark Energy Cosmology: The Equivalent Description via Different Theoretical Models and Cosmography Tests. *Astrophysics and Space Science, 342,* 155-228.

17 Copeland, E.J., Sami, M., & Tsujikawa, S. (2006). Dynamics of dark energy. *International Journal of Modern Physics D, 15*(11), 1753–1935.

18 Capozziello, S., De Laurentis, M., Nojiri, S., & Odintsov, S.D. (2009). Classifying and avoiding singularities in the alternative gravity dark energy models. *Physical Review D, 79*, 559.

19 Ribas, M.O., Devecchi, F.P., & Kremer, G.M. (2005). Fermions as sources of accelerated regimes in cosmology. *Physical Review D, 72*, 3502.

20 Samojeden, L.L., Devecchi, F.P., & Kremer, G.M. (2010). Fermions in Brans-Dicke cosmology. *Physical Review D, 81*, 7301.

21 Samojeden, L.L., Kremer, G.M., & Devecchi, F.P. (2009). Accelerated expansion in bosonic and fermionic 2D cosmologies with quantum effects. *Europhys. Letters, 87*, 1.

22 Razina, O., Tsyba, P., Meirbekov, B., & Myrzakulov, R. (2019). Cosmological Einstein-Maxwell model with g-essence. *International Journal of Modern Physics D, 28*(10), 1950126.

23 Razina, O., Tsyba, P., & Sagidullayeva, Z.M. (2019). Power solution of the f (R)-gravity with Maxwell term and g-essence. *Bulletin of the university of Karaganda-Physics, 1*(93), 94–102.

24 Ribas, M.O., Zambianchi, P., Devecchi, F.P., & Kremer, G.M. (2012). Fermions in a Walecka-type cosmology. *Europhysics Letters, 97*, 5.

25 Kulnazarov, I., Yerzhanov, K., Razina, O., Myrzakul, Sh., Tsyba, P., & Myrzakulov, R. (2011). G-essence with Yukawa Interactions. *The European Physical Journal C, 71*(7), 1698.

26 Tsyba, P., Razina, O., Barkova, Z., Bekov, S., & Myrzakulov, R. (2019). Scenario of the evolution of the universe with equation of state of the Weierstrass type gas. *Journal of Physics: Conference Series, 1391*, 012162.

27 Razina, O.V., Tsyba, Yu. P., Myrzakulov, R., Meirbekov, B., & Shanina, Z. (2019). Cosmological Yang-Mills model with k-essence. *Journal of Physics Conference Series, 1391*, 012164.

28 Altayeva, G., Razina, O., & Tsyba, P. (2022). Cosmography in the multifield cosmological model. *Bulletin of the University of Karaganda-Physics, 2*(106), 37-48.

29 Myrzakulova, S.A., Razina, O.V., Myrzakulov, N.A., & Altaybaeva, A.B. (2022). Slow-roll inflation in the k-essence model with a periodic scalar field function. *Recent Contributions to Physics, 81*(2), 19-27.

30 Rakhatov, D.Z., Tsyba, P.Y., & Razina, O.V. (2022). Power-law solutions of the f(G) gravity model with electromagnetic and scalar field. *Recent Contributions to Physics, 80*(1), 12-21.

31 Razino, O., Myrzakulov, Y., Serikbayev, N., Myrzakul, S., Nugmanova, G., & Myrzakulov, R. (2011). G-essence cosmologies with scalar-fermion interactions. *European Physical Journal Plus, 126*(9), 85.

32 Bolotin, Yu.L., Erokhin, D.A., & Lemets, O.A. (2012). Rasshiriaiushchaiasia Vselennaia: zamedlenie ili uskorenie? [Expanding Universe: slowdown or speedup?]. *Uspekhi fizicheskikh nauk — Advances in the physical sciences, 55* (9), 876–918 [in Russian].

33 Arjona, R., & Nesseris, S. (2021). Novel null tests for the spatial curvature and homogeneity of the Universe and their machine learning reconstructions. *Physical Review D, 103,* 063537.

34 Capozziello, S., Cardone, V.F., & Salzano, V. (2008). Cosmography of f (R) gravity. *Physical Review D, 78*, 063504.

35 Capozziello, S., D'Agostino, R., & Luongo, O. (2008). Gravity Cosmography. *General Relativity and Quantum Cosmology, 1*(35), 82.