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Z.K. Shanina, O.V. Razina

L.N. Gumilyov Eurasian National University, Nur-Sultan, Kazakhstan
(E-mail: shaninazk@gmail.com)

Modified models of a free boson string and their solutions

In this paper we consider the modified models of a free boson string and examine their dynamics. Modified models of the boson string are investigated from the point of view of the second order formalism (the Polyakov action). The motion of the free string propagates in Minkowski space-time. The internal geometry of the free boson string is described by the metric $h_{\alpha\beta}$ having the signature of Minkowski. The classical string trajectory is described by the function X^{μ}. The introduction of the function K and the potential $V(X^{\mu})$ into the action of the boson string allows us to find new approaches to solving the problem of the boson string dynamics. For these modified models of the boson string the equations of motion are found. The equations of motion are obtained by varying the action with respect to X^{μ}. The energy-momentum tensor for considered models of the free boson string is calculated. Constraint conditions are obtained for all types of modified boson string models. It is proved that constraint conditions are satisfied for each considered modified model of the boson string. Solutions of the equations of motion for each modified boson string model are also considered and obtained. String's world sheet has been built for these solutions. It is shown that all solutions for these models satisfy the equations of motion and the constraint conditions.

Keywords: bozon string, action, equation of motion, energy-momentum tenzor, constraint conditions.

1 Introduction

Despite its successes, the Standard model of elementary particles must be built into a broader theory that would include gravity as well as strong and electroweak interactions. Currently, there is only one likely candidate for such a theory: string theory, which emerged in the 1960s as a not-so-successful model of hadrons and only later emerged as a possible theory of all interactions [1].

String theory touches on the deepest questions of the universe and is the most developed modern attempt to answer questions about the nature of fundamental interactions.

String models appeared in physics in the sixties and seventies of the XX century in attempts to describe the mechanism of strong interactions [2]. Strings are one-dimensional extended objects with a length of the order of 10^{-35} m, which is not a subject to observation at present. Their dynamics is considered on a two-dimensional surface — a world sheet — in the background manifold (n-dimensional space-time) and described geometrically. Bosonic models have meaning of twenty-six dimensions and their fluctuations are bosons, and fermion and superstring — in ten dimensions [3].

The boson string theory is a fundamental theory and most important concepts of string theory can be explained within this theory. Boson string models are considered and studied in [4, 5]. Generalized models of boson strings with potential and non-canonical kinetic term were considered in [6, 7].

Despite the numerous studies available in the field of boson string theory, this theory is not fully studied and requires further development. The need to study different models of boson strings is motivated by the study of the unification of all the interactions in nature. The study of bosonic string theory can be considered equivalent to the study of two-dimensional gravity associated with scalars.

The article described the bosonic string model and equations of motion; calculated tensor of energy-momentum for these models; the solution for the bosonic string. Conclusions are drawn.

2. Models of bosonic string

The action for the freely moving string can be constructed similarly to the action of a point particle. In the simplest case, the action is described by its d-dimensional Minkowski coordinates $X^{\mu}(\sigma,\tau)$. The parameters σ and τ set the points on the world sheet that the string sweeps when it moves; σ is the coordinate along the space-like direction and τ is along the time-like direction [8].

Let's introduce the metric $h_{\alpha\beta}$ and the inverse metric $h^{\alpha\beta}$ $(\alpha,\beta=0,1)$ on the world sheet. The action has the form

$$S=-\frac{T}{2}\int d\tau\int d\sigma\sqrt{-h}K, \tag{1}$$

where $h=det(h_{\alpha\beta})$, T is the constant factor equal to string tension, K is the some function of its arguments. Consider several models for different K.

2.1. Model $K=F(Y)$

$$K=F(Y);$$
$$Y=h^{\alpha\beta}\partial_\alpha X^\mu\partial_\beta X_\mu=h^{00}[\partial_0X^\mu\partial_0X_\mu]+h^{11}[\partial_1X^\mu\partial_1X_\mu]=h^{00}\dot{X}^{\mu2}+h^{11}X'^{\mu2}, \tag{2}$$

where the dot means the derivative with respect to τ and the prime — derivative with respect to σ.

Components of metric taken the form

$$h_{\alpha\beta}=\begin{pmatrix}h_{00} & h_{01}\\ h_{10} & h_{11}\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}. \tag{3}$$

We rewrite the action (1) as

$$S=-\frac{T}{2}\int d\tau\int d\sigma\sqrt{-h}F(Y). \tag{4}$$

In order to obtain the equations of motion of the boson string, we find the variation of action (4) with respect to X^μ

$$\delta S=-\frac{T}{2}\int d\tau\int d\sigma\sqrt{-h}\delta F(Y)=\frac{T}{2}\int d\tau\int d\sigma\sqrt{-h}F_Y[h^{00}\ddot{X}^\mu+h^{11}X''^\mu]\delta X^\mu.$$

Applying (3) we write down the equation of motion

$$F_Y(\ddot{X}^\mu-X''^\mu)=0. \tag{5}$$

The energy-momentum tensor of two-dimensional field theory is written as

$$T_{\alpha\beta}=-\frac{2}{T}\frac{1}{\sqrt{-h}}\frac{\delta S}{\delta h^{\alpha\beta}}=0. \tag{6}$$

In order to find the energy-momentum tensor, we vary action (4) with respect to $h^{\alpha\beta}$. We find the energy-momentum tensor for the considered model

$$\delta S_{h^{\alpha\beta}}=-\frac{1}{2}T\int\int d\tau d\sigma\left[\delta\sqrt{-h}F(Y)+\sqrt{-h}\delta F(Y)\right]. \tag{7}$$

The following formulas are useful for varying the action

$$\delta h=-hh_{\alpha\beta}\delta h^{\alpha\beta}, \tag{8}$$

it follows that

$$\delta\sqrt{-h}=-\frac{1}{2}\sqrt{-h}h_{\alpha\beta}\delta h^{\alpha\beta} \tag{9}$$

and

$$\delta F=F_Y\delta Y=F_YY_{h^{\alpha\beta}}\delta h^{\alpha\beta}. \tag{10}$$

Therefore (7) will take the form

$$\delta S_{h^{\alpha\beta}}=-\frac{1}{2}T\int\int d\tau d\sigma\sqrt{-h}\left[-\frac{1}{2}h_{\alpha\beta}F+F_YY_{h^{\alpha\beta}}\right]\delta h^{\alpha\beta}. \tag{11}$$

Substituting (11) into (6) we obtain the energy-momentum tensor

$$T_{\alpha\beta}=F_YY_{h^{\alpha\beta}}-\frac{1}{2}h_{\alpha\beta}F=F_Y\partial_\alpha X^\mu\partial_\beta X_\mu-\frac{1}{2}h_{\alpha\beta}F.$$

Since the metric on the world sheet has been calibrated, the vanishing of the energy-momentum tensor, i.e. $T_{\alpha\beta}=0$, based on the equation of motion of the world sheet metric, should be introduced as additional constraint conditions

$$T_{\alpha\beta}=F_Y\partial_\alpha X^\mu\partial_\beta X_\mu-\frac{1}{2}h_{\alpha\beta}F=0. \tag{12}$$

Taking into account (2) and (3) we write constraint conditions (12)

$$T_{00} = F_Y \dot{X}^2 - \frac{1}{2} F = 0; \tag{13}$$

$$T_{11} = F_Y X'^2 + \frac{1}{2} F = 0; \tag{14}$$

$$T_{01} = T_{10} = F_Y \dot{X} X' = 0. \tag{15}$$

From (13)–(15) follow that constraint conditions

$$\dot{X}^2 + X'^2 = 0;$$

$$\dot{X} X' = 0.$$

In relativistic string theory, these conditions are called orthonormal calibration.

Equation of motion (5) can have several solutions. Let's consider some models of these solutions.

2.1.1. Model $X^\mu = A_\mu \cos(\tau + \sigma)$.

$$\dot{X}^\mu = -A_\mu \sin(\tau + \sigma);$$

$$\ddot{X}^\mu = -A_\mu \cos(\tau + \sigma) = -X^\mu.$$

$$X'^\mu = -A_\mu \sin(\tau + \sigma);$$

$$X''^\mu = -A_\mu \cos(\tau + \sigma) = -X^\mu.$$

Constraint conditions for this model

1) $$\dot{X}^2 + X'^2 = 2A_\mu^2 \sin^2(\tau + \sigma) = 0;$$

$$A_\mu \neq 0 \Rightarrow \sin^2(\tau + \sigma) = 0;$$

$$\tau + \sigma = \pi n, n \in Z.$$

2) $$\dot{X} \cdot X' = A_\mu^2 \sin^2(\tau + \sigma) = 0;$$

$$\tau + \sigma = \pi n, n \in Z.$$

Substitute the solutions into the equations of motion (5)

$$F_Y[-A_\mu \cos(\tau + \sigma) + A_\mu \cos(\tau + \sigma)] = 0.$$

2.1.2. Model $X^\mu = A_\mu \cos(\tau - \sigma)$.

$$\dot{X}^\mu = -A_\mu \sin(\tau - \sigma);$$

$$\ddot{X}^\mu = -A_\mu \cos(\tau - \sigma) = -X^\mu.$$

$$X'^\mu = A_\mu \sin(\tau - \sigma);$$

$$X''^\mu = -A_\mu \cos(\tau - \sigma) = -X^\mu.$$

We write constraint conditions for this model

1) $$\dot{X}^2 + X'^2 = 2A_\mu^2 \sin^2(\tau - \sigma) = 0;$$

$$A_\mu \neq 0 \Rightarrow \sin^2(\tau - \sigma) = 0;$$

$$\tau - \sigma = \pi n, n \in Z.$$

2) $$\dot{X} \cdot X' = -A_\mu^2 \sin^2(\tau - \sigma) = 0;$$

$$\tau - \sigma = \pi n, n \in Z.$$

This model satisfies the equation of motion (5)

$$F_Y[-A_\mu \cos(\tau - \sigma) + A_\mu \cos(\tau - \sigma)] = 0.$$

2.1.3. Model $X^\mu = A_\mu \cos[B_\mu(\tau - \sigma)]$.

$$\dot{X}^\mu = -A_\mu B_\mu \sin[B_\mu(\tau - \sigma)];$$

$$\ddot{X}^\mu = -A_\mu B_\mu^2 \cos[B_\mu(\tau - \sigma)].$$

$$X'^{\mu} = A_{\mu}B_{\mu}\sin[B_{\mu}(\tau-\sigma)];$$

$$X''^{\mu} = -A_{\mu}B_{\mu}^2\cos[B_{\mu}(\tau-\sigma)].$$

Constraint conditions for the model 2.1.3 take a form

1) $$\dot{X}^2 + X'^2 = 2A_{\mu}^2B_{\mu}^2\sin^2[B_{\mu}(\tau-\sigma)] = 0;$$

$$\sin^2[B_{\mu}(\tau-\sigma)] = 0;$$

$$\tau-\sigma = \frac{\pi n}{B_{\mu}}, n\in Z.$$

2) $$\dot{X}\cdot X' = -A_{\mu}^2B_{\mu}^2\sin^2[B_{\mu}(\tau-\sigma)] = 0;$$

$$\tau-\sigma = \frac{\pi n}{B_{\mu}}, n\in Z.$$

We substitute this model also into the equation of motion (5)

$$F_Y[-A_{\mu}B_{\mu}^2\cos(\tau-\sigma) + A_{\mu}B_{\mu}^2\cos(\tau-\sigma)] = 0.$$

From the three examples, it can be seen that all considered models are solutions of the equation of motion (5).

2.2. Model $K = F(Y)\cdot V(X^{\mu})$.

We write the action (1) as

$$S = -\frac{T}{2}\int d\tau\int d\sigma\sqrt{-h}F(Y)\cdot V(X^{\mu}). \quad (16)$$

To obtain the equations of motion of the boson string, we find the variation of action (16) with respect to X^{μ}

$$\delta S = -\frac{T}{2}\int d\tau\int d\sigma\sqrt{-h}\delta(F(Y)\cdot V(X^{\mu})) =$$

$$= -\frac{T}{2}\int d\tau\int d\sigma\sqrt{-h}[-2F_Y\cdot V(h^{00}\ddot{X}^{\mu} + h^{11}X''^{\mu}) + F\cdot V_{X^{\mu}}]\delta X^{\mu}.$$

Applying (3) we write down the equation of motion

$$F_YV(\ddot{X}^{\mu} - X''^{\mu}) - \frac{1}{2}FV_{X^{\mu}} = 0. \quad (17)$$

We find the variation of action by $h^{\alpha\beta}$

$$\delta S_{h^{\alpha\beta}} = -\frac{1}{2}T\int\int d\tau d\sigma V(X^{\mu})\left[\delta\sqrt{-h}F(Y) + \sqrt{-h}\delta F(Y)\right]. \quad (18)$$

Using (8)–(10) write the action

$$\delta S_{h^{\alpha\beta}} = -\frac{1}{2}T\int\int d\tau d\sigma V(X^{\mu})\sqrt{-h}\left[-\frac{1}{2}h_{\alpha\beta}F + F_YZ_{h^{\alpha\beta}}\right]\delta h^{\alpha\beta}. \quad (19)$$

Substituting (19) into (6) we obtain the energy-momentum tensor

$$T_{\alpha\beta} = V[F_YZ_{h^{\alpha\beta}} - \frac{1}{2}h_{\alpha\beta}F] = V[F_Y\partial_{\alpha}X^{\mu}\partial_{\beta}X_{\mu} - \frac{1}{2}h_{\alpha\beta}F].$$

The equation of motion (17) must be supplemented by the constraint conditions $T_{\alpha\beta} = 0$.

$$T_{\alpha\beta} = V[F_Y\partial_{\alpha}X^{\mu}\partial_{\beta}X_{\mu} - \frac{1}{2}h_{\alpha\beta}F] = 0. \quad (20)$$

Taking into account (2) and (3) we write the constraint conditions (20)

$$T_{00} = V(F_Y\dot{X}^2 - \frac{1}{2}F) = 0; \quad (21)$$

$$T_{11} = V(F_YX'^2 + \frac{1}{2}F) = 0; \quad (22)$$

$$T_{01} = T_{10} = F_YV\dot{X}X' = 0. \quad (23)$$

From (21)-(23) follow constraint conditions

$$\dot{X}^2 + X'^2 = 0;$$
$$\dot{X}X' = 0.$$

We consider some solution models of the equation of motion (17).

2.2.1. Model $X^\mu = A_\mu \cos(\tau + \sigma)$.

$$\dot{X}^\mu = -A_\mu \sin(\tau + \sigma);$$
$$\ddot{X}^\mu = -A_\mu \cos(\tau + \sigma) = -X^\mu.$$
$$X'^\mu = -A_\mu \sin(\tau + \sigma);$$
$$X''^\mu = -A_\mu \cos(\tau + \sigma) = -X^\mu.$$

The constraint conditions for the given model are

1) $$\dot{X}^2 + X'^2 = 2A_\mu^2 \sin^2(\tau + \sigma) = 0;$$
$$A_\mu \neq 0 \Rightarrow \sin^2(\tau + \sigma) = 0;$$
$$\tau + \sigma = \pi n, n \in Z.$$

2) $$\dot{X} \cdot X' = A_\mu^2 \sin^2(\tau + \sigma) = 0;$$
$$\tau + \sigma = \pi n, n \in Z.$$

Substitute given model in the equation of motion (17)

$$F_Y V[-A_\mu \cos(\tau + \sigma) + A_\mu \cos(\tau + \sigma)] - \frac{1}{2} F V_{X^\mu} = 0;$$
$$F V_{X^\mu} = 0.$$
$$F \neq 0, V_{X^\mu} = 0, V = const.$$

In space, the string can move as you like and in its motion in space-time sweeps the surface, called the world sheet of the string, which is a two-dimensional surface with spatial σ and temporal τ coordinates. Moving in *d*-dimensional space-time, the string oscillates. In order for these oscillations not to contradict quantum mechanics and special relativity theory, the number of space-time dimensions for the boson string must be 26.

Figure 1 shows the distribution of the boson string world sheet in coordinates τ and σ for model 2.2.1.

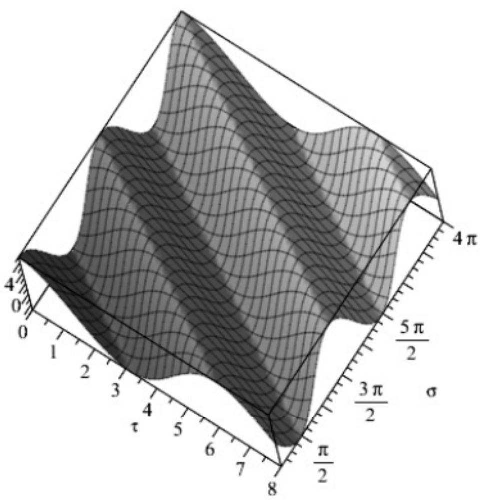

Figure 1. The world sheet of a modified bosonic strings for the solution $X^\mu = A_\mu \cos(\tau + \sigma)$ at $A_\mu = 4$

If the string is closed, then among other oscillations in the spectrum of its motion will be a particle with zero mass and spin 2, the so-called graviton — which is the carrier of the gravitational interaction.

In curved spacetime, a closed string experiences curvature as it moves. In this case, in order not to violate the laws of quantum theory, curved spacetime must be a solution to Einstein's equations.

2.2.2. Model $X^{\mu} = A_{\mu}\cos(\tau-\sigma)$.

$$\dot{X}^{\mu} = -A_{\mu}\sin(\tau-\sigma);$$
$$\ddot{X}^{\mu} = -A_{\mu}\cos(\tau-\sigma) = -X^{\mu}.$$
$$X'^{\mu} = A_{\mu}\sin(\tau-\sigma);$$
$$X''^{\mu} = -A_{\mu}\cos(\tau-\sigma) = -X^{\mu}.$$

Constraint conditions for the model 2.2.2 can be written as

1) $$\dot{X}^2 + X'^2 = 2A_{\mu}^2\sin^2(\tau-\sigma) = 0;$$
$$A_{\mu} \neq 0 \Rightarrow \sin^2(\tau-\sigma) = 0;$$
$$\tau-\sigma = \pi n, n \in Z.$$

2) $$\dot{X}\cdot X' = -A_{\mu}^2\sin^2(\tau-\sigma) = 0;$$
$$\tau-\sigma = \pi n, n \in Z.$$

Substitute this model in the equation of motion (17)

$$F_Y V[-A_{\mu}\cos(\tau-\sigma) + A_{\mu}\cos(\tau-\sigma)] - \frac{1}{2}FV_{X^{\mu}} = 0;$$
$$FV_{X^{\mu}} = 0.$$
$$F \neq 0, V_{X^{\mu}} = 0, V = const.$$

Figure 2 shows the distribution of the boson string world sheet in coordinates τ and σ for model 2.2.2.

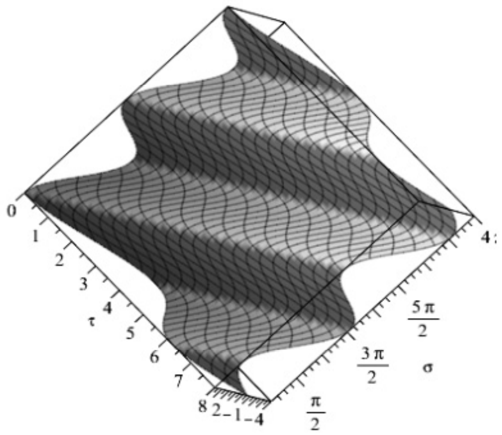

Figure 2. The world sheet of a modified bosonic strings for the solution $X^{\mu} = A_{\mu}\cos(\tau-\sigma)$ at $A_{\mu} = 4$

2.2.3. Model $X^{\mu} = A_{\mu}\cos[B_{\mu}(\tau-\sigma)]$.

$$\dot{X}^{\mu} = -A_{\mu}B_{\mu}\sin[B_{\mu}(\tau-\sigma)];$$
$$\ddot{X}^{\mu} = -A_{\mu}B_{\mu}^2\cos[B_{\mu}(\tau-\sigma)].$$
$$X'^{\mu} = A_{\mu}B_{\mu}\sin[B_{\mu}(\tau-\sigma)];$$
$$X''^{\mu} = -A_{\mu}B_{\mu}^2\cos[B_{\mu}(\tau-\sigma)].$$

The constraint conditions for this model

1) $$\dot{X}^2 + X'^2 = 2A_{\mu}^2B_{\mu}^2\sin^2[B_{\mu}(\tau-\sigma] = 0;$$
$$\sin^2[B_{\mu}(\tau-\sigma)] = 0;$$
$$\tau-\sigma = \frac{\pi n}{B_{\mu}}, n \in Z.$$

$$\dot{X}\cdot X' = -A_\mu^2 B_\mu^2 \sin^2[B_\mu(\tau-\sigma)] = 0;$$

2)

$$\tau-\sigma = \frac{\pi n}{B_\mu}, n\in Z.$$

Substitute solutions in the equation of motion (17)

$$F_Y V\left[-A_\mu B_\mu^2 \cos[B_\mu(\tau-\sigma)] + A_\mu B_\mu^2 \cos[B_\mu(\tau-\sigma)]\right] - \frac{1}{2} F V_{X^\mu} = 0;$$

$$F V_{X^\mu} = 0.$$

$$F \neq 0, V_{X^\mu} = 0, V = const.$$

Figure 3 shows the distribution of the boson string world sheet in coordinates τ and σ for model 2.2.3.

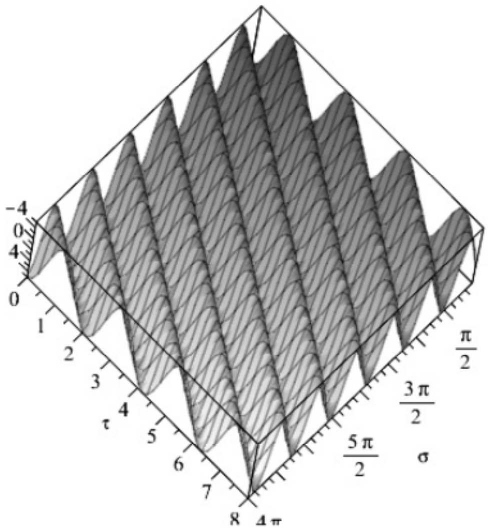

Figure 3. The world sheet of a modified bosonic strings for the solution $X^\mu = A_\mu \cos[B_\mu(\tau-\sigma)]$ at $A_\mu = 5, B_\mu = 3$

All considered models are solutions of the equation of motion (20).

3. Conclusion

In the article the dynamics of the bosonic string for various modified string models was studied. Considered actions for given bosonic models of strings and the equations of motion are obtained by varying the action with respect to X^μ; calculated tensor of energy-momentum for these models, obtained the constrait conductions; solutions obtained for the modified models of bosonic strings. Built world sheets of bosonic strings for some solutions. It is checked that the constrait conditions for all considered models are satisfied.

The same classical string theory may have different quantum versions of their models. Some of them may have a non-physical dimension of space-time and tachyon states. Others may be formulated in ordinary four-dimensional spacetime and have no tachyon states. Theoretical physicists are interested in both groups of quantum string models. The appearance of relativistic strings in four-dimensional space-time is currently considered in cosmology as the most likely candidate for explaining the process of appearance of inhomogeneities in the distribution of matter in the early universe, which caused the appearance of galaxies. Therefore, the study of various models describing the dynamics of the boson string, will make a choice in favor of the most adequate models.

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З.К. Шанина, О.В. Разина

Еркін бозондық ішектің модификацияланған модельдері және олардың шешімдері

Мақалада еркін бозондық ішектің модификацияланған модельдері қарастырылды және олардың динамикасы зерттелді. Бозондық ішектің модификацияланған модельдері екінші реттік формализм (Поляков) тұрғысынан зерттелген. Еркін ішектің қозғалысы Минковский кеңістік-уақытында таралады. Еркін бозондық ішектің ішкі геометриясы Минковский сигнатурасына ие $h_{\alpha\beta}$ метрикамен сипатталды. Ішектің классикалық траекториясы X^{μ} функциямен ұсынылған. Бозондық ішектің әсеріне K функциясы мен $V(X^{\mu})$ потенциалды енгізу бозондық ішек динамикасының мәселелерін шешуде жаңа тәсілдер табуға мүмкіндік береді. Бозондық ішектің берілген модификацияланған модельдері үшін қозғалыс теңдеулері табылды, олар X^{μ} әсеріне қатысты вариациялау әдісі бойынша алынды. Еркін бозондық ішектің қарастырылатын модельдері үшін энергия-импульс тензоры есептелді. Бозондық ішек модельдерінің барлық түрлері үшін байланыс шарттары алынды. Әрбір қарастырылатын модель үшін байланыс шарттарының орындалатындығы дәлелденді. Сонымен қатар, бозондық ішектің модификацияланған модельдері үшін қозғалыс теңдеулерінің шешімдері қарастырылды және алынды. Берілген шешімдер үшін ішектің әлемдік парағы құрылды. Берілген модельдер үшін барлық шешімдер қозғалыс теңдеулерін және байланыс шарттарын қанағаттандыратындығы көрсетілді.

Кілт сөздер: бозондық ішек, әсер, қозғалыс теңдеуі, энергия-импульс тензоры, байланыс шарттары.

З.К. Шанина, О.В. Разина

Модифицированные модели свободной бозонной струны и их решения

В статье рассмотрены модифицированные модели свободной бозонной струны и исследована их динамика. Модифицированные модели бозонной струны изучены с точки зрения формализма второго порядка (действие Полякова). Движение свободной струны распространяется в пространстве-времени Минковского. Внутренняя геометрия свободной бозонной струны описывается метрикой $h_{\alpha\beta}$, имеющей сигнатуру Минковского. Классическая траектория струны представлена функцией X^{μ}. Введение в действие бозонной струны функции K и потенциала $V(X^{\mu})$ позволяет найти новые подходы к решению проблемы динамики бозонной струны. Для данных модифицированных моделей бозонной струны найдены уравнения движения, которые получены методом варьирования действия относительно X^{μ}. Вычислен тензор энергии-импульса для рассматриваемых моделей свободной бозонной струны. Получены условия связей для всех видов модифицированных моделей бозонной струны. Доказано, что условия связей выполняются для каждой рассматриваемой модифицированной модели бозонной струны. Также изучены полученные решения уравнений движения для каждой модифицированной модели бозонной струны. Построен мировой лист струны для данных решений. Показано, что все решения для данных моделей удовлетворяют уравнениям движения и условиям связей.

Ключевые слова: бозонная струна, действие, уравнение движения, тензор энергии-импульса, условия связей.

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